AN INVESTIGATION OF CONTROLLED FREE SURFACE EFFECT ON FIN STABILIZATION OF A VESSEL HAVING A LARGE METACENTRIC HEIGHT

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ABSTRACT

AN INVESTIGATION OF A CONTROLLED FREE SURFACE EFFECT ON FIN STABILIZATION OF A VESSEL HAVING A LARGE METACENTRIC HEIGHT

by

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Submitted to the Department of Naval Architecture and Marine Engineering on May 26th, 1958, in partial fulfillment of the requirements for the degree of Naval Engineer and the degree of Master of Science in Naval Architecture and Marine Engineering.

The objective of this study was to investigate the feasibility of reducing the cost, size and weight of an activated fin stabilizer system of a ship with a large metacentric height, by the introduction of a free surface effect. The free surface causes a virtual reduction in the ship's metacentric height, and consequently lowers the requirements of the fin stabilization system.

Calculations for a typical large warship indicated that the free surface effect could reduce the cost by about one-third, and the weight of the overall stabilization system by about 22%. Space requirements were essentially unchanged. Achievement of these reductions, however, required the extensive use of broad, shallow tanks. It proved to be impractical to install these tanks in the existing ship. The use of wrap-around tanks was investigated; but because of the effects of resonance between tank fluids and waves these tanks were found to be dynamically incompatible with an activated fin stabilization system.

In addition to the problem of tank location, the damage stability of the ship was greatly impaired by the free surface. The resulting condition of damage stability is considered sufficient in itself to rule out the proposed installation. Therefore, the installation of such a system is not recommended for general use in ships. Limited application of the system may be feasible, but only when it is an integral part of the initial design of the ship.

Thesis Supervisor: Philip Mandel

Title: Associate Professor of Naval Architecture

Cambridge 39, Massachusetts
May 26, 1958

Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

In accordance with the requirements for the Degree of Naval Engineer and the Degree of Master of Science in Naval Architecture and Marine Engineering, we herewith submit a thesis entitled: "An Investigation of a Controlled Free Surface Effect on Fin Stabilization of a Vessel Having a Large Metacentric Height."

Respectfully,

CHARLES G. KOSONEN
Lieutenant, United States Navy

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I. INTRODUCTION

The problem of roll stabilization of ships is one that has received considerable attention over a number of years. Many different means of achieving roll stabilization have been proposed and tried. Among the more successful of these have been systems using passive and active tanks, active gyros and movable fins.

Tanks containing liquids were first used as a means of roll quenching by Sir Philip Watts in an installation in HMS Inflexible in 1883 [38] [39]. The system comprised several water chambers which were one deck height high and extended from one side of the ship to the other. The chambers were partially filled with water, which provided a free surface effect. This resulted in a lowering of metacentric height and a lengthening of the ship's period of roll. The purpose of the system was to decrease the large angles of roll experienced when the frequency of wave encounter was in synchronism with the ship's frequency of rolling. This was achieved partly by the reduction in metacentric height, which decreased the effective righting arm, and partly by the increase in ship's period, which shifted the occurrence of resonance to a longer and less often encountered wave length. Tests in still water and among waves showed an appreciable reduction of rolling amplitude. The use of this system was not continued as the water chambers occupied considerable space which was needed for other purposes.

In 1911, Herr H. Frahm introduced the U-tube tank, which was designed to achieve the same purpose, that of diminishing the extreme angles of roll that occur when a ship is rolling in synchronism with the waves [12]. This tank operated on a secondary resonance between the motion of the fluid in the tank and the ship's rolling motion. The tuning of the tank

to the period of the ship resulted in a phase lag of the fluid in the tank such that the center of gravity of the fluid was always on the rising side of the ship, thus absorbing energy on each roll. The system was successful in the band of wave encounter frequencies near resonance; however, at frequencies somewhat removed from resonance the effect of the phase lag was to increase the amplitude of roll [35].

Both the Watts water chambers and the Frahm U-tube tanks were passive systems, and as such were able to effect a relatively small degree of stabilization. They depended on the rolling motion of the ship for their effectiveness and could not completely stabilize the ship. Further, according to Minorsky,

> "Onboard the low stability ships built at the beginning of this century, and under the conditions of a regular pattern of waves encountered, the performance of passive tanks was found to be satisfactory and roll quenching of the order of 50% was frequently reported. In later years, with the advent of ships of higher metacentric height and in the usual generally confused seaway, the behavior of passive tanks was found to be so unsatisfactory that this method of stabilization was gradually discontinued shortly after the first world war." [26]

In more recent years a higher degree of stabilization has been sought, and the means of achieving this has been by the use of active systems, which predict the motions of the ship and act to produce a stabilizing moment which counteracts the inclining moment. The active gyro is one example of this type of system which proved capable of fairly effective stabilization, but which exacted a high cost in space, weight and money. The currently popular method of roll stabilization is the use of activated fins, which, in a normal ship, can accomplish a high degree of stabilization at a reasonable cost in weight and space.

The capacity of a roll stabilizing system is defined as the angle to which the system can incline the vessel in still water. For a system to

incline a vessel to an angle ϕ, the system must generate an inclining moment equal to $\triangle GM \sin \phi$, where $\triangle$ represents the ship's displacement, GM it's metacentric height, and ϕ is held to small values, on the order of 7 degrees or less. From this definition it can be seen that for a ship of given displacement the torque producing ability of a stabilizing system of given capacity will vary directly with the ship's metacentric height. The size and weight of the system may be expected to increase with it's torque producing ability. Thus, on a ship with a large metacentric height, such as a large warship, the stabilizing system may be quite large and heavy.

The need for roll stabilization of large warships is increasing with the development of missile systems which require a stable platform for launching; and yet, the requirement for a stabilizing system of reasonable size, low metacentric height, appears to be incompatible with other requirements, that is sufficient metacentric height to insure safety after damage.

It is well known that the effect of loose liquids within a ship is a virtual decrease in the metacentric height. It is the purpose of this investigation to determine whether or not the cost, size and weight of the required fin stabilizing system for a typical large warship with a high metacentric height can be reduced by the purposeful introduction of a free surface effect. It is hoped to accomplish this with no appreciable impairment of the vessel's damage stability.

The free surface installation is initially envisaged as comprising a number of the ship's already existing wing fuel oil or ballast tanks, connected athwartships so as to form wrap-around or U-tube tanks. This might be done at a relatively low increase in weight and volume, the

only additions being the piping or ducting necessary to effect the cross-over connection. The ship's fuel oil or ballast would provide the free surface and no increase in weight would be caused by the introduction of extra liquids.

The investigation will comprise the following steps:

1. A study of the dynamic compatibility of the proposed free surface with fin stabilization.
2. A study of the required shape and dimensions of tankage.
3. An approximate design of fin and free surface stabilizing system.
4. A study of the ship's intact and damage stability with the free surface installed, and of means of restoring metacentric height when desired.
5. A comparison of the weights and volumes of the fin and free surface system with those of a fin system without free surface installed.

II. PROCEDURE

A. Dynamic Compatibility of the Passive Tank with an Activated Fin System

Froude introduced the theory that any body floating among waves is acted upon by the same forces as those which would have acted upon the water displaced by the body. Confirmation of this theory has been obtained through experiments. The water in a wave is acted upon by forces which have a resultant force perpendicular to the wave surface. This resultant force is equal to the gravitational force. Therefore, the apparent gravity acting on floating body among waves is normal to the effective wave slope [22].

In order to investigate the effect of free surface on the requirements of an activated fin stabilization system, the moments acting upon a ship with large free surface effect (Ship A) will be compared with those moments acting upon a ship having negligible free surface effects (Ship B). The two ships, Ship A and Ship B, are to be identical in all other respects.

In this comparison, several simplifying assumptions will be made as follows:

1) both ships are assumed to be perfectly stabilized; i.e., the ships remain vertical at all times.

2) the effective wave slope is essentially the same as the surface waveslope. From trochoidal wave theory the maximum wave slope at the surface is

$$\phi_o = \tan^{-1} \frac{r_o}{R} \qquad (1)$$

The effective waveslope passes approximately through the center of bouyancy of the ship and the maximum effective waveslope is

$$\phi_o = \tan^{-1} \frac{r_1}{R} \qquad (2)$$

Substituting the relationship,

$$r_1 = r_o e^{-2\pi h/\lambda} \tag{3}$$

where h is the distance from the surface to the center of buoyancy and λ is the wave length.

$$\tan \phi_1 = \frac{r_o}{R} e^{-2\pi h/\lambda} \tag{4a}$$

or

$$\tan \phi_1 = e^{-2\pi h/\lambda} \tan \phi_o \tag{4b}$$

Therefore, where h = 10 feet, an error of less than 10% will exist in assuming the effective waveslope is equal to the surface waveslope, if

$$\frac{2\pi h}{\lambda} < 0.105 \quad \text{or} \quad \lambda > 600 \text{ feet}$$

In the Atlantic for normally rough weather, the maximum effective wave slope is of the order of $5^o - 6^o$ [25]. Thus, the error in this assumption is about $1/2^o$.

3) the wave length is much greater than the breadth of ship, i.e., the wave surface is essentially flat across the breadth of the ship.

4) the wave motions are essentially sinusoidal,

5) the direction of advance of the waves is normal to the longitudinal axis of the ship.

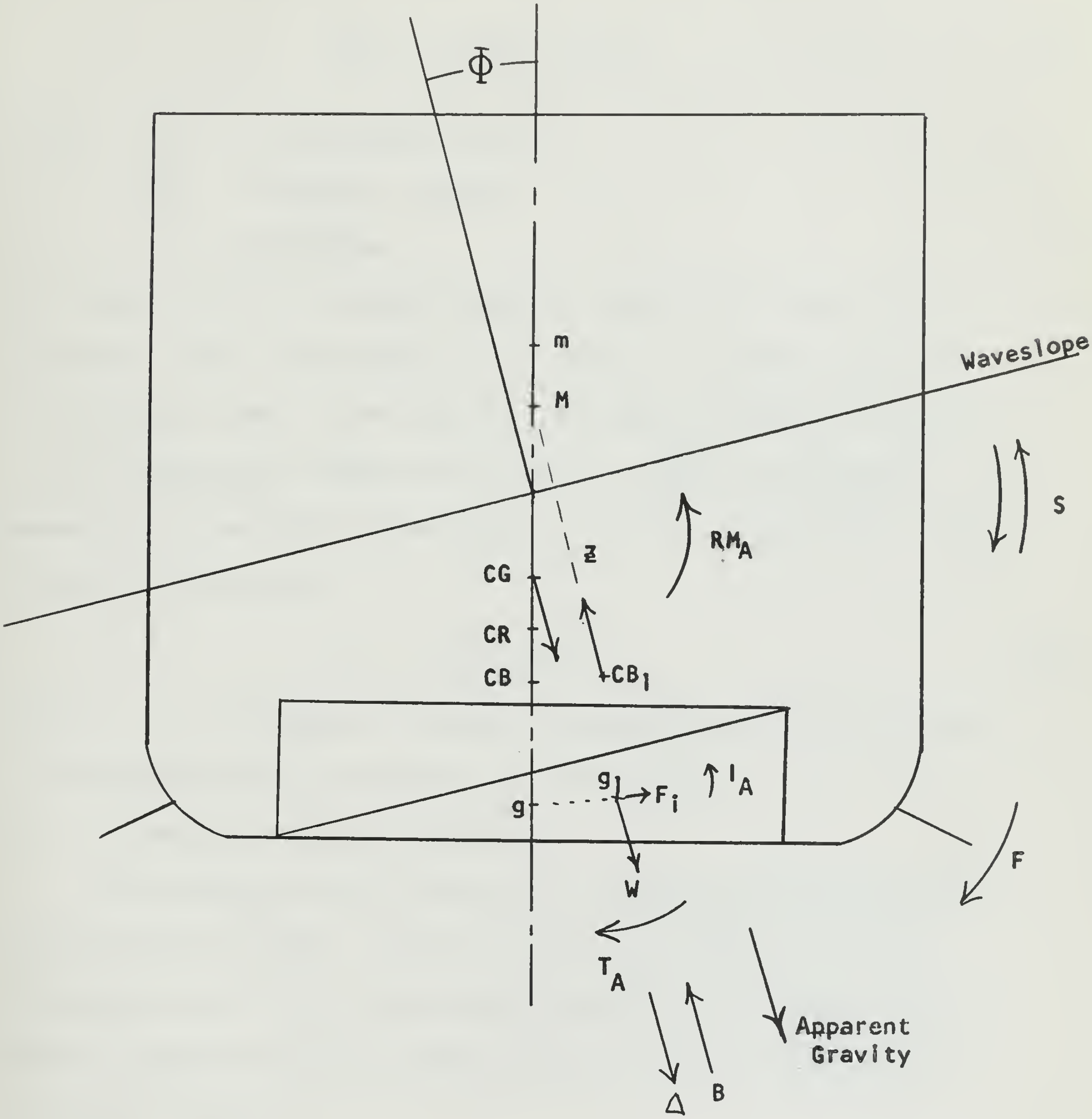
6) the coupling of motions other than roll into a rolling moment is neglected.

7) the still water rolling is isochronous.

First, consider the forces and moments acting upon Ship A when it is at the maximum waveslope (See Figure 1). This point is selected because the maximum moments will be generated at this location. Since

FIGURE I

Forces and Moments Acting on a Stabilized Ship in Waves



the apparent gravity is normal to the waveslope the righting moment due to the couple formed by the gravity and buoyancy forces is equal to,

$$RM_A = \triangle \, GM \sin \phi \tag{5}$$

Where, $\triangle$ = displacement of ship

GM = metacentric height

ϕ = waveslope

The liquid in the free surface tanks will seek a level parallel to the waveslope. The liquid (either oil or water) will behave essentially the same as an ideal fluid since the restriction of flow is negligible in a box shaped tank without swash plates or other interferences. The moment due to the shift in the center of gravity of the liquid in the tanks will be equal to,

$$T_A = \triangle \frac{i}{V} \sin \phi \tag{6}$$

where, i = the moment of inertia of the surface of the liquid about its longitudinal axis, corrected for fluid density.

V = volume displacement of the ship.

The shifting liquid in the tank will also generate a moment due to the inertial effects. For the present this moment will be represented simply as I_A. This inertial moment may be either positive or negative depending upon the tank location relative to the center of rotation of the ship.

The sea will exert a moment resisting the roll of the ship. This moment will be, in the main, caused by the frictional resistance of the water on the wetted surface of the hull, by the wave-making effect of the rotating ship (rotating relative to the wave surface), and by the acceleration of entrained water. The resisting moment of the sea,

S_A, will be a function of $\frac{d\phi}{dt}$, $\left(\frac{d\phi}{dt}\right)^2$, and $\frac{d^2\phi}{dt^2}$, and may be calculated by empirical methods [22] [13].

The required moment of the fin system, F_A, is equal to the sum of the other moments,

$$F_A = \triangle GM \sin \phi - \frac{i}{V} \sin \phi + S_A \pm I_A \tag{7}$$

In ship B the required moment of the fin system will be greater by,

$$\delta F = \triangle GG_v \sin \phi \pm I_A \tag{8}$$

where GG_V is the virtual rise of the ship's center of gravity due to the free surface effect.

B. Investigation of the Tuning Effect in a Wrap-Around Tank

In the initial stages of the investigation it was proposed to install the free surface in the form of a wrap-around or U-tube system. This form of tank would permit using the ship's installed fuel oil or ballast wing tanks, with a crossover channel through the double bottom to effect the athwartships connection. Such an arrangement would allow the greatest reduction in metacentric height with the least surface area required, in view of the outboard location of the tanks, and would require no infringement on spaces that might be used for other purposes.

A study of this type of tank configuration showed that any tank of such nature is, in effect, a tuned tank. Any restriction to the flow from one side to the other, such as the proposed crossover channel, results in the assignment of a definite period of oscillation to the fluid in the tank. In a tuned tank the natural period of oscillation, [35]

$$T_t = 2\pi\sqrt{\frac{1}{2g}\int\frac{A}{A_o}\,dl} \tag{9}$$

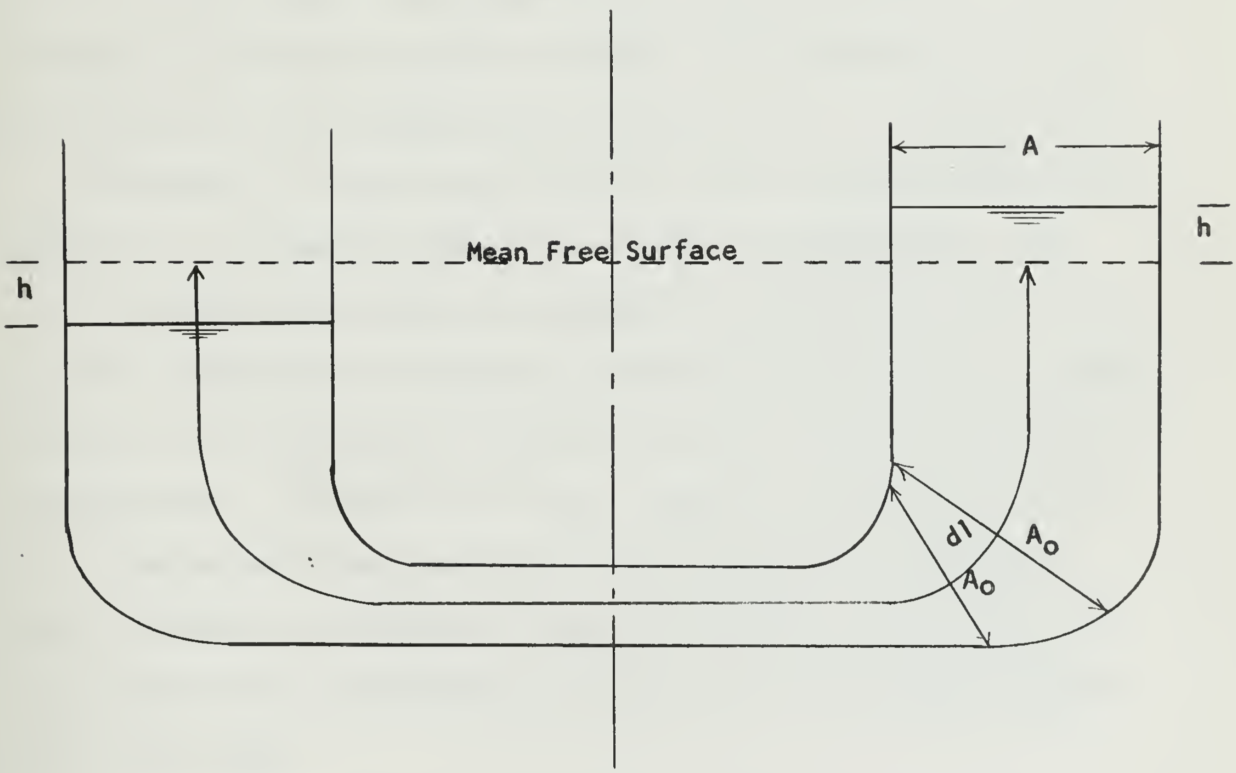
where g is the acceleration due to gravity, A is the area of the mean free surface on one side, A_o is the cross-sectional area normal to the flow of fluid at any point and dl is an increment of length along the flow path. Figure 11 illustrates the symbology used.

It should be noted that even if A_o equals A throughout the length of the tank a definite period is still assigned and that only as $\int\frac{A}{A_o}\,dl$ becomes small does the period diminish appreciably.

In an installation where a ship is stabilized by a tuned tank, the period of the tank is adjusted to a value somewhat lower than the

FIGURE II

<u>Cross Section of a U-Tube Tank</u>



natural period of rolling of the ship. [35] [43]. Then, in regular seas, an oscillation is set up within the tank which tends to oppose the rolling motion of the ship. This opposition to rolling is the result of a 90° phase lag between the motion of the tank fluid and the incidence of the waves, which causes the center of gravity of the tank fluid to be always on the side of the ship which is rolling upwards, thus absorbing energy at all times. The effect is most prominent when the period of wave encounter is in near synchronism with the period of the tank. At wave frequencies on either side of this band however, the phase difference is such as to cause an increase in the amplitude of rolling over that which would be experienced without the tank in operation. [25].

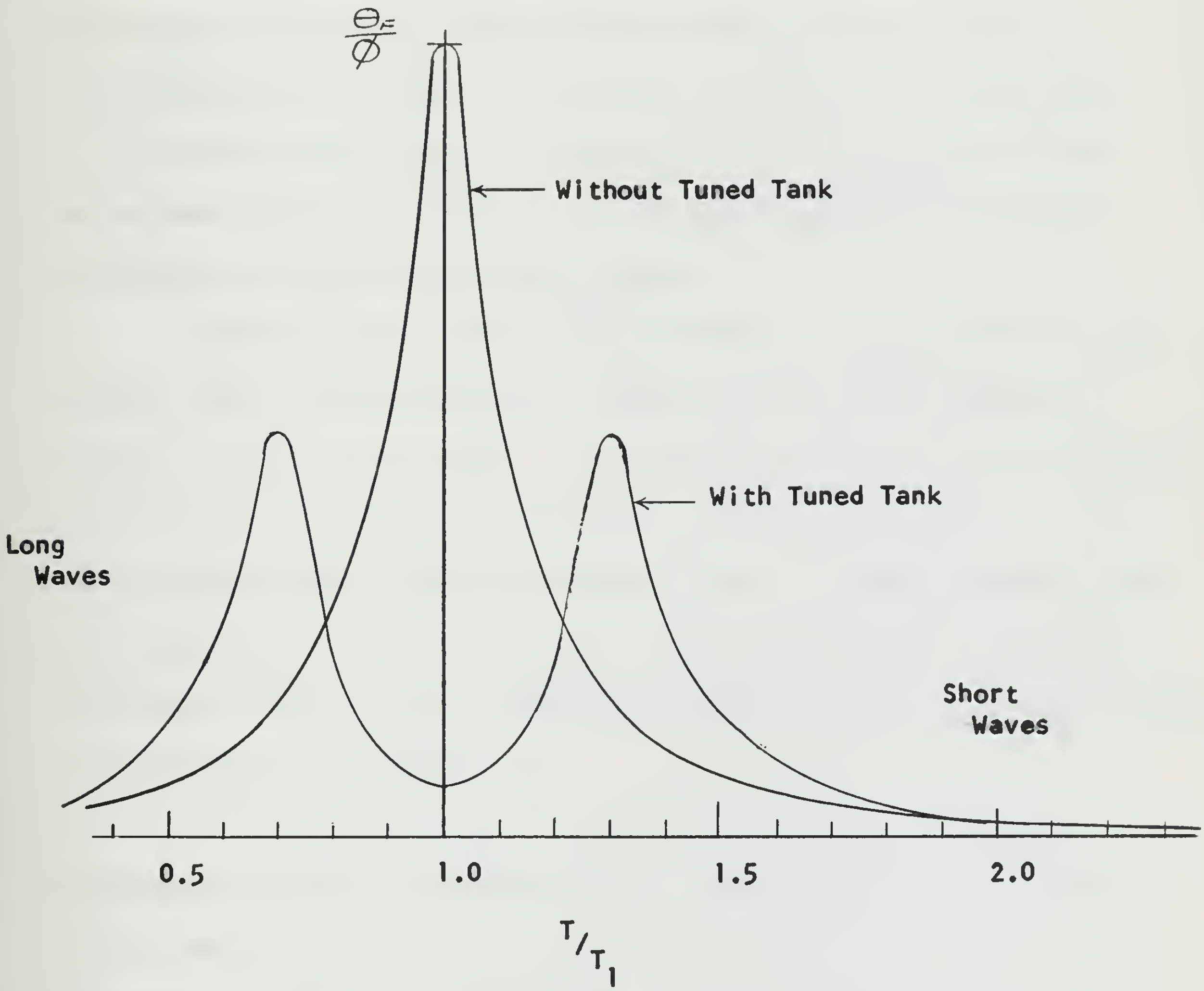
The effect of a tuned tank on rolling can be shown diagramatically by Figure III, a diagram presented by a number of authorities. [35].

It should be noted that this type of tank was developed for ships which had a relatively low metacentric height and tended to roll in their own period. With this type of ship the tuned tank was fairly effective in reducing roll. In ships with higher metacenter, which tend to roll more in the period of the waves, the effectiveness of the tank is greatly lessened, as the ship is rolling at effective frequencies much less of the time and the desired oscillatory motion of the tank fluids is not set up. [25].

It must be emphasized that the effectiveness of the tuned tank is the result of the center of gravity of the fluid staying on the rising side of the ship. This is not the motion which would be followed by a fluid with a true free surface; instead the center of gravity of the fluid would be always on the low side of a rolling ship, while that side is both falling and rising, or on the side deepest immersed in

FIGURE III

Resonance Effect of Tuned Tanks on Roll Amplitude



θ = Maximum angle of forced oscillation

ϕ = Maximum angle of wave slope

T = Natural period of ship

T_1 = Period of wave encounter

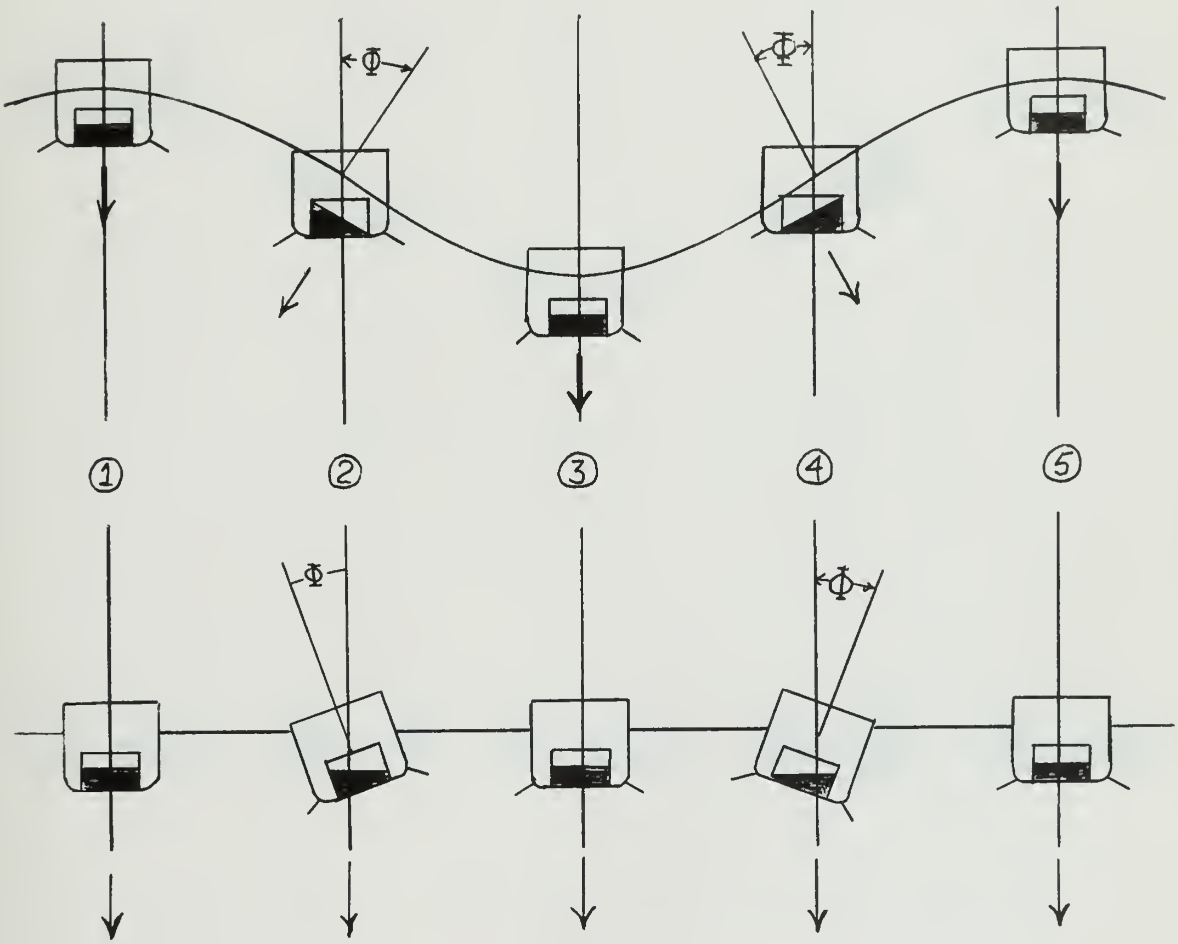
the wave slope if the ship remained vertical. While the tuned tank may be effective at some frequencies in quenching roll, it has yet to be shown whether or not such fluid motion is beneficial in a fin stabilized ship. It should be borne in mind that there exists a basic difference in the philosophy of design of the Frahm U-tube tank and the stabilizing system under investigation. The former was a system designed to diminish the large angles of roll experienced when the ship was rolling in synchronism with the waves. The purpose of the current investigation is to determine the effects of free surface on a fin stabilized ship, which experiences very little roll regardless of the wave frequency, within the limits of the system's capacity.

To facilitate the study of these motions an analogy will be drawn between a ship remaining vertical among waves and a ship undergoing forced rolling in still water. If one considers the motion of a perfectly stabilized ship in waves, it can be seen that the ship appears to roll, relative to the wave surface, to an angle equal to the maximum wave slope. Since the apparent gravity is normal to the wave surface, this motion is analogous to that of a ship undergoing forced rolling of the same amplitude in still water. See Figure IV.

To illustrate the effects of fluid motion in a tuned tank two extreme conditions will be discussed; one, a vessel having a large metacentric height, such as a raft, which would tend to remain level with the waveslope, and the other a stabilized ship, which remains vertical regardless of the wave slope. The raft, remaining level with the waveslope, would be analogous to a vessel not rolling at all in still water. See Figure V. The effect of the apparent gravity, in forcing the fluid to seek a level parallel to the waveslope, results in no fluid motion

FIGURE IV

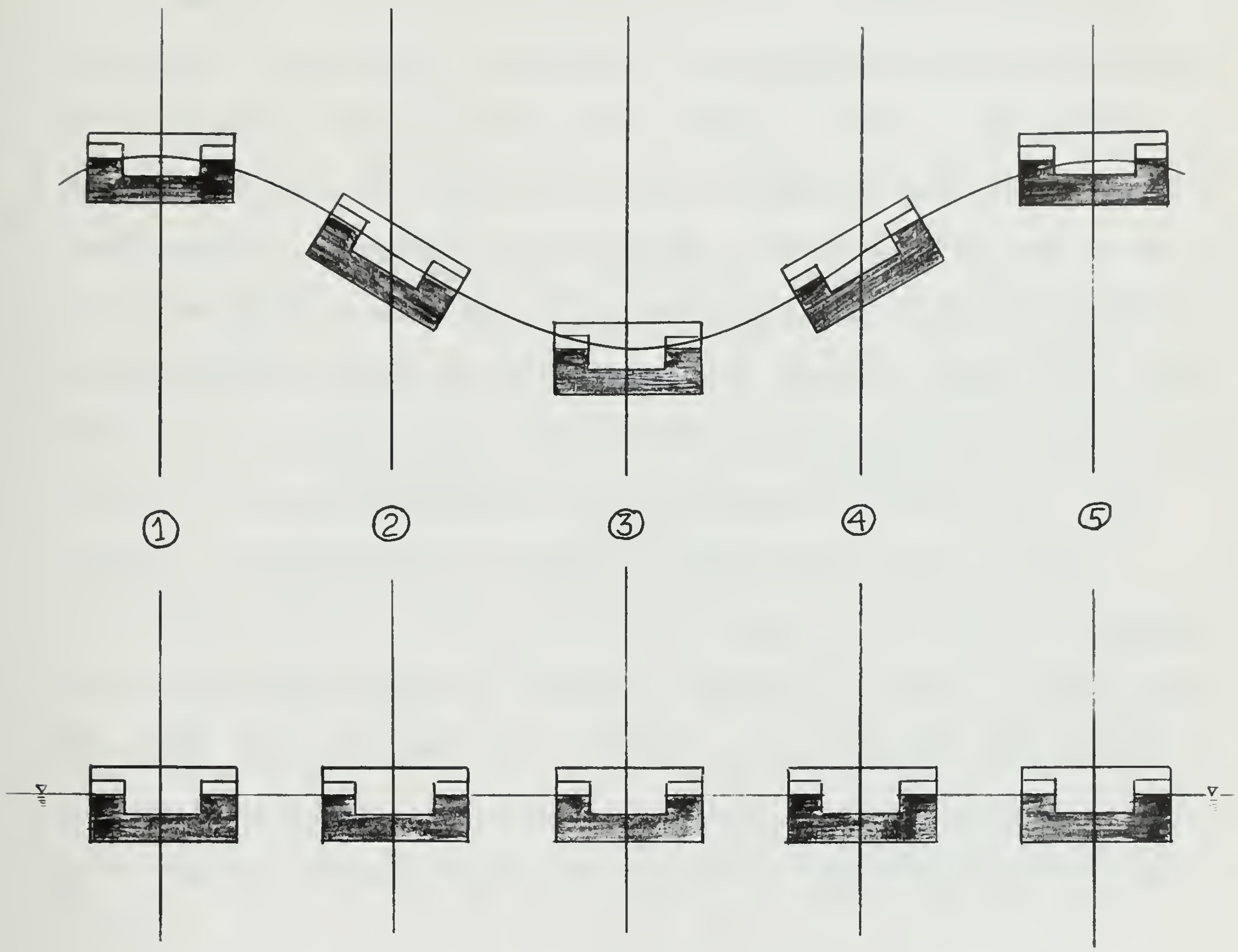
Still Water Rolling Analogy of a Stabilized Ship in Waves



Note: Arrows indicate direction of apparent gravity

FIGURE V

Action of a Passive Tank in a Raft Type Ship



Note: Liquid remains at mean level at all times

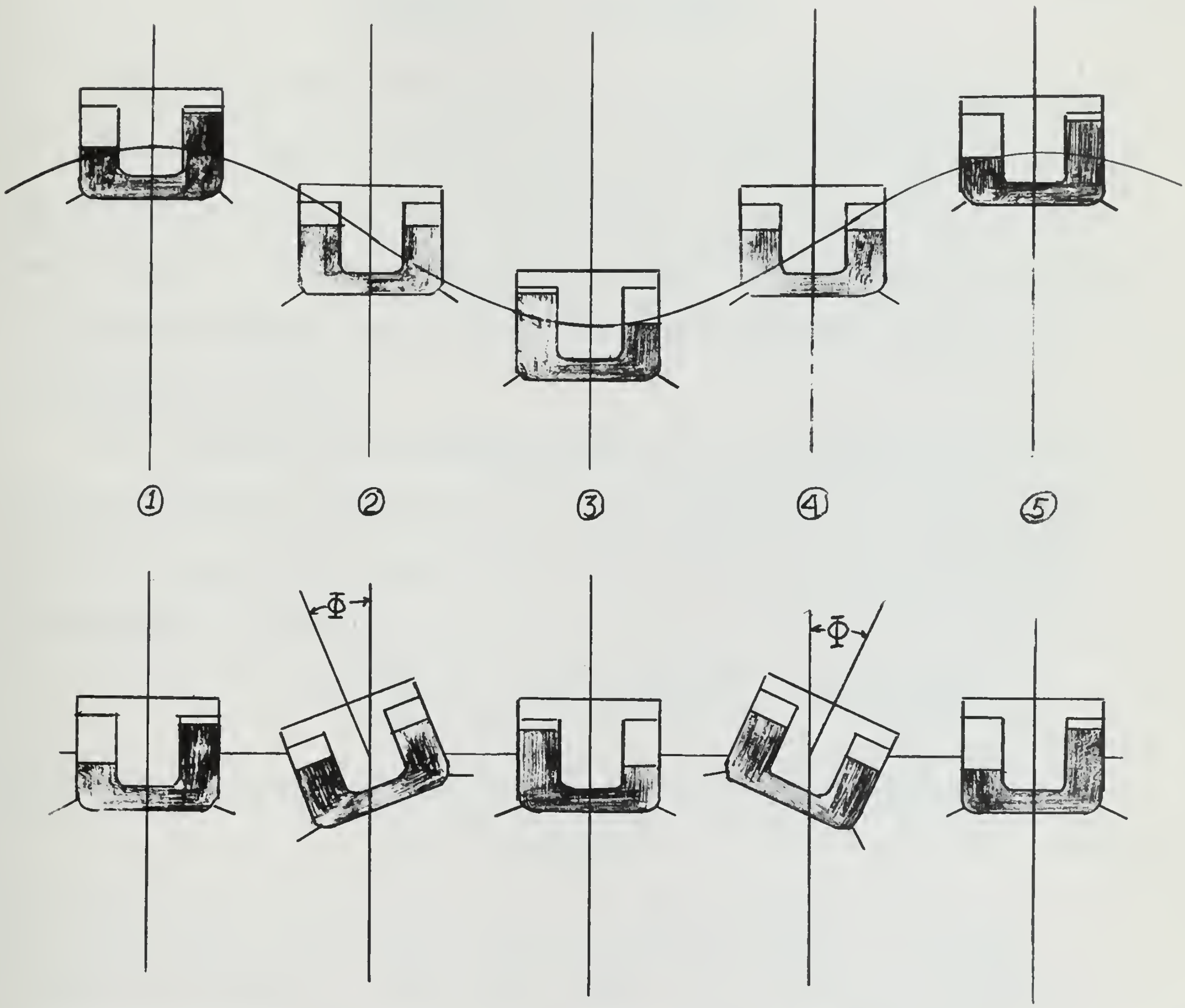
relative to the vessel, thus completing the still water analogy. In this instance there is no inclining moment generated by the fluid in the tanks.

The perfectly stabilized ship, remaining vertical among the waves, is analogous to a ship rolling in still water to the amplitude of the waveslope. Here, again, the effect of the constantly shifting direction of the apparent force of gravity will cause the fluid to seek the level of the waveslope, or, in the still water analogy, to seek the horizontal. See Figure VI. If there is a restriction to the fluidflow, such as exists in a tuned tank, a phase lag will be introduced, which will reach 90° when the tank period and the wave period are in synchronism. [22]. Thus, in this condition of synchronism the fluid introduces a moment which tends to force the ship to a position normal to the waveslope, or, in the still water analogy, the moment would be such as to oppose the forced rolling. It can be seen that if, in the still water analogy, a fin stabilizer system were being used to force the rolling, the action of the tank fluids would oppose the action of the fins. Similarly, in a ship stabilized by fins and remaining upright among waves, the action of the fluid would be such as to require a greater torque from the fins to maintain the upright attitude.

Thus, in any fin stabilized vessel having a tuned tank, if the natural frequency of the tank is within the range of wave incidence frequencies likely to be encountered, the tuned tank will exert its maximum detrimental moment at this wave frequency and thus decrease the effectiveness of the fin system. At other wave frequencies there will still be a detrimental effect, although a somewhat lesser one, because of the phase lag introduced by the restriction to flow.

FIGURE VI

Action of a Passive Tuned Tank in a Fin Stabilized Ship



Note: There is a 90° phase lag of liquid in tank

Robb [35] has stated that waves about 500 feet long are the limit of those commonly observed. Current wave theory holds that storm waves are made up of a spectrum of wave lengths which includes both very long waves (over 1000 feet) and very short waves [44]. Further, the apparent wave length, L_o, may be increased if the ship is proceeding at a speed, V (knots) on a course at an angle, α, to the direction of advance of the waves [22].

$$L_o = \frac{L_w}{2.26\sqrt{L_w} - 1.69 V \cos\alpha} \qquad (10)$$

Thus, the period of the exciting force of the waves may be very long.

Figure VII shows a cross-section of the typical large warship selected to illustrate the stabilizing system proposed. In this ship a fuel oil or ballast tank is located in the position shown in the figure. The remainder of the double bottom is void space. The space inboard of the voids is allocated to machinery.

If it assumed that the machinery space may be encroached upon so as to make a cross-over channel equal in area throughout its length to the area of the mean free surface on one side of the tank, the tank period, from equation (9) would be

$$T_t = 2\pi\sqrt{\frac{1}{2g} \times 2\,(5.15 + 39.75)} = 7.45 \text{ seconds.}$$

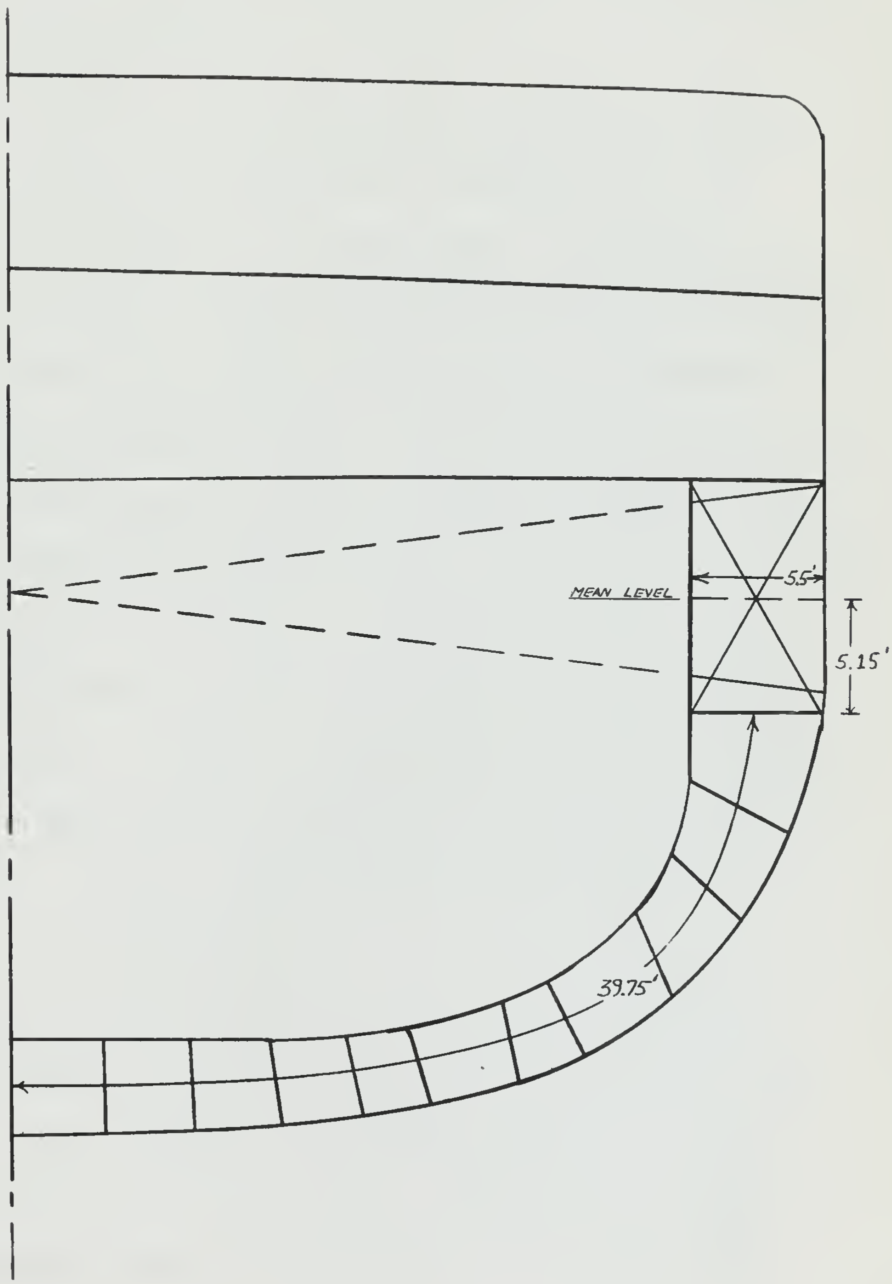
Wave length and wave period are related by the following formula [29]:

$$T_w = .44\sqrt{L_w} \qquad (11)$$

Thus, the proposed tank would reach resonance when waves of 286 feet length were encountered. This wave length is well within the limits of waves commonly encountered and the tank would therefor exert a

FIGURE VII

Cross Section of a Typical Large Warship



detrimental influence on the action of the fin system.

It can be seen from equation (9) that to decrease the period of the tank appreciably, approximating the free surface, would require an inordinately large cross-channel area. To avoid the effects of resonance the cross-channel area would have to be so great as to eliminate any advantage in space requirements that the U-tube tank has over the true free surface tank.

The period of the tank may be increased by reducing the cross-channel area, thus shifting the occurrence of resonance to a wave length long enough that resonance may rarely occur. For example, if the cross channel area were one-tenth the mean free surface area throughout the length, the tank period would be 22.5 seconds and the wave length for resonance would be 2550 feet. With this period however, in waves of a shorter period, such as would be commonly encountered, the restriction to flow would be so great that very little transfer of fluid weight would occur. The force of gravity tending to cause a shift of fluid would be reversed too quickly for any effective weight transfer to get started. Figure III shows that the effect of fluid in a tuned tank is practically negligible when the tank period is much greater than the wave period. In this condition the fluid acts nearly as though it were in a solid flooded tank.

From these arguments it is concluded that a wrap-around of U-tube tank is not capable of producing the desired free surface effect and that, instead, it can generate a detrimental moment, increasing the requirements on a fin stabilizer system. To obtain the desired reduction in metacentric height an uninterrupted free surface tank may be installed.

The true free surface tank also has a natural period of oscillation, and, under conditions of resonance, may generate a detrimental moment. The motion of the center of gravity of the fluid is defined by the following equation:

$$\frac{d^2\phi}{dt^2} + a_g \frac{gm}{k^2} \phi = 0 \tag{12}$$

and the period of the tank is then given by,

$$T_t = 2\pi\sqrt{\frac{k^2}{a_g \ gm}} \tag{13}$$

where $k^2 = \frac{i}{lb} = \frac{lb^3}{12 \ lb}$ (14)

and gm is the height of the metacenter of the fluid,

$$gm = \frac{i}{lbd} = \frac{lb^3}{12 \ lbd} \tag{15}$$

Equation (13) then becomes,

$$T_t = 2\pi\sqrt{\frac{lb^3 \ 12 \ lbd}{12 \ lb \ a_g \ lb^3}} = 2\pi\sqrt{\frac{d}{a_g}} \tag{16}$$

In the above derivation:

l is the length of the tank

b is the width of the tank

d is the depth of fluid in the tank

k is the radius of gyration of the fluid surface

i is the moment of inertia of the fluid surface about its longitudinal centerline

a_g is the acceleration due to gravity

Reference to Appendix A will show that the depth of fluid in the tank decided upon for this investigation is 4.22 feet. With such a depth the period of the tank,

$$T_t = 2\pi\sqrt{\frac{4.22}{32.2}} = 2.28 \text{ seconds.}$$

A wave having a period, T_w, will have a length,

$$L_w = \left(\frac{T_w}{0.44}\right)^2 \qquad (17)$$

Thus, the corresponding wave length for resonance in this tank will be,

$$L_w = \left(\frac{2.28}{0.44}\right)^2 = 26.7 \text{ feet.}$$

It can be seen that waves of this length represent only 37.7% of the ship's beam and may have a height of about 4.45 feet if the extreme height to length ratio of 1/6 is applied. The relative size of the wave and the ship are indicated in Figure IX.

The force which causes the motion of the fluid in a stabilized ship, as shown in section II-A, is the force of apparent gravity, a resultant of true gravity and the centrifugal force acting on a vessel undergoing an orbital motion in the waves. With a wave length as short as 26.7 feet and a beam almost three times this value the orbital motion will not exist, the waves being virtually ripples on the ship's profile. The apparent gravity will, to all intents and purposes, continue to act in a vertical direction and there will be no force tending to cause the tank fluid to shift. Accordingly, the condition of resonance which might cause a detrimental moment will not occur; and it is concluded that the true free surface tank is capable of providing the desired reduction in metacentric height.

C. Approximate Design of Fin and Free Surface Stabilizing Installations

As a basis of comparison of two stabilizing systems, one consisting of fin stabilization alone and the other consisting of both fins and a free surface, a fin stabilizer system was designed for the typical warship selected as it now exists. A capacity of 5 degrees at 15 knots was chosen as representing good practice. The methods of calculation are similar to those used in references [5] and [19]. The calculations for this system will be found in Appendix A. The fin system arrived at consisted of three pairs of fins, each fin having a span of 12 feet and a chord length of 5.79 feet. The assumption was made that the fins could be longitudinally displaced sufficiently so that no reduction in lift would occur from mutual interference. This would require a spacing of 10 chord lengths between the fins on a side or a total length of installation of about 133 feet, which is feasible in this ship. The effective lever arm of the fins was based on an average location and the selected span.

To effect the reduction which is the purpose of this investigation an arbitrary decision was reached to install a free surface which would allow stabilization of the same capacity with only two sets of fins. This was implemented by a one-third reduction on the torque producing capability of the whole fin system.

It has been shown in equation (8) that there exists in a free surface tank both a static and a dynamic moment. The static moment is the result of a transverse shift in the center of gravity of the fluid in the tank and may be expressed as a reduction in the ship's virtual metacentric height. The dynamic moment is a result of the

acceleration imparted to the fluid in moving from side to side. This moment may be positive, negative or zero, depending on the vertical location of the tank relative to the ship's center of rotation.

In the calculation of the tankage necessary to accomplish the desired reduction in the fin stabilizer installation the dynamic moment, I_A, will be considered negligible. Justification for this may be seen in the following comparison of I_A with thestatic moment, T_A.

The static force causing the moment T_A about the center of rotation of the ship is simply the weight of the liquid in the tank. Referring to Figure VIII this is shown as W and is equal to $\rho_w lbd$ where the dimension l is the longitudinal length of the tank. This force acts through the arm gm sin $\emptyset$. From this, the moment, T_A, equals:

$$T_A = W \; gm \; \sin \emptyset \tag{18a}$$

$$= \rho_w lbd \; gm \; \sin \emptyset \tag{18b}$$

and since;

$$gm = i/v = \frac{lb^3/12}{lbd} = \frac{b^2}{12\,d} \tag{19}$$

then

$$T_A = \frac{\rho_w lb^3 \sin \emptyset}{12} \tag{20}$$

The motion of the center of gravity of the liquid is assumed to follow the laws of simple harmonic motion with the period of the exciting force equal to the period of the waves encountered, therefore;

$$\emptyset = \emptyset_{max} \sin \omega t \tag{21}$$

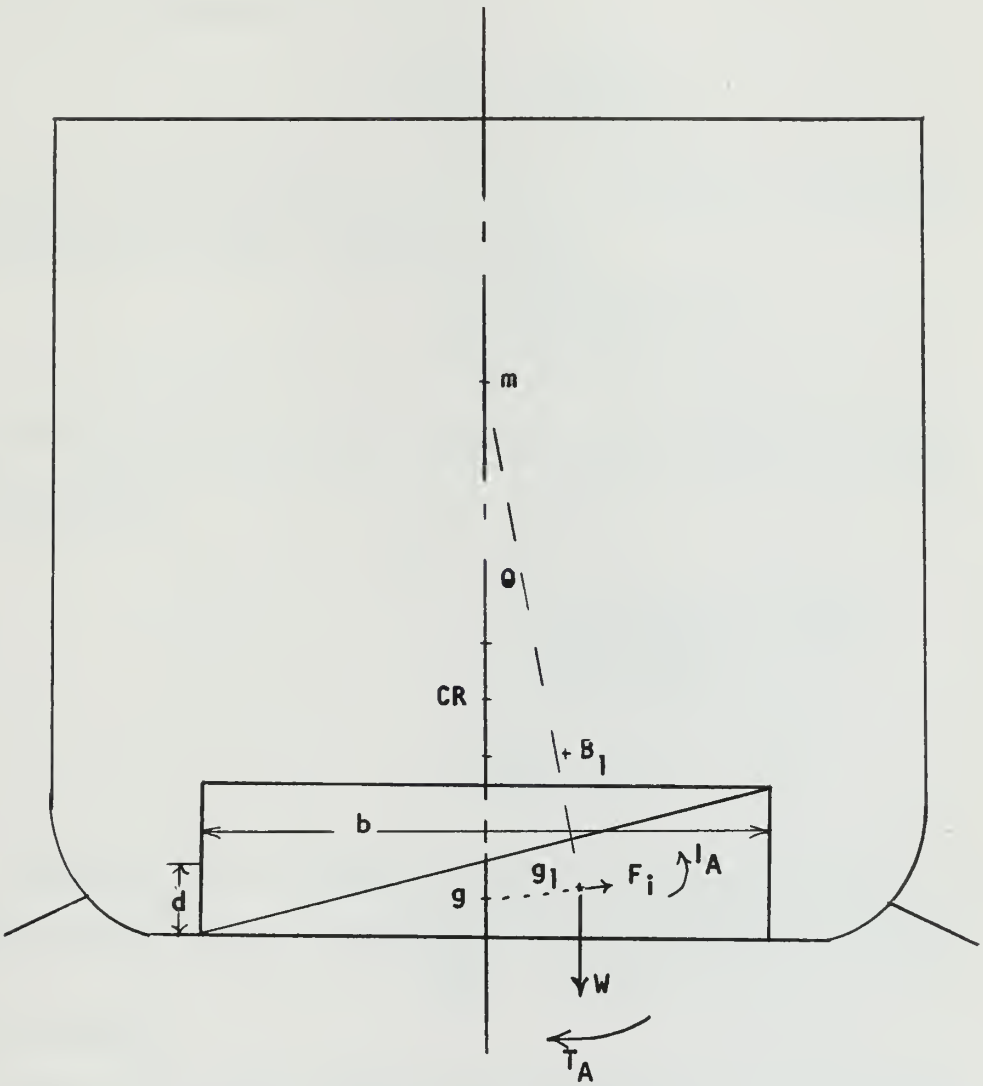
and

$$\frac{d^2\emptyset}{dt^2} = - \emptyset_{max} \, \omega^2 \sin \omega t \tag{22}$$

Further, since to neglect the inertia effect the maximum value must be considered, the maximum inertia force occurs when the acceleration

FIGURE VIII

Inertial Effect in Free Surface Tank



is a maximum, or when;

$$\frac{d^2\phi}{dt^2} = - \phi_{max} \omega^2 \tag{23}$$

From this the maximum inertia force, F_i, is

$$F_i = \frac{W}{a_g} gm \frac{d^2\phi}{dt^2}_{max} \tag{24}$$

where a_g is the acceleration due to gravity, and gm is the radius about which the liquid in the tank rotates.

$$F_i = \rho_w \frac{lbd\ b^2\ \phi_{max}\ \omega^2}{a_g\ 12\ d} \tag{25}$$

Then the moment caused by this force about the center of rotation of the ship is;

$$I_A = F_i\ \overline{CRg} \tag{26a}$$

$$= \rho_w \frac{lbd\ b^2\ \phi_{max}\ \omega^2\ \overline{CRg}}{a_g\ 12\ d} \tag{26b}$$

$$= \rho_w \frac{lb^3}{12a_g}\ \phi_{max}\ \omega^2\ \overline{CRg} \tag{26c}$$

Comparing the two moments, T_A and I_A, it is seen that I_A differs from T_A by the factor $\frac{\overline{CRg}\ \omega^2}{a_g}$ for the values of ϕ that are of interest.

The value of this factor may be determined with a knowledge of the geometry of the ship and the frequency of waves likely to be encountered.

In the case in question a wave period of 11 seconds, which corresponds to a wave having a length of approximately 600 feet, is assumed as being representative. This yields;

$$\omega^2 = \left(\frac{2\pi}{T}\right)^2 = 0.296 \tag{27}$$

The value of $\overline{CR_g}$ is determined by the distance between the center of rotation of the ship and the center of gravity of the free surface tank. In the type of ship being considered, which is typical of many large warships, the center of rotation is approximately 21 feet above the base line. The longitudinal strength requirements of the vessel prohibit placing the tank directly on the bottom of the ship. This restriction requires that the tank bottom be at least 4 feet above the base line. The distance from the bottom of the tank to the center of gravity of the liquid is, of course, a function of the tank dimensions. In the proposed free surface tank this distance is greater than 2 feet. As a conservative estimate the distance $\overline{CRg}$ is then set at 15 feet. Then the entire factor is

$$\frac{\overline{CRg}\ \omega^2}{a_g} = \frac{15}{32.2} \times 0.296 = 13.8\% \tag{28}$$

While at first glance this appears to be a significant part of the moment generated by the tank, it must be remembered that in every consideration the worst possible values were assigned. Probably the most presumptuous of these is the likelihood that this tank will be located at the lowest part of the ship. While good practice indicates such a location from the point of view of keeping the center of gravity low, practically such a location will seldom be possible due to the requirements on these spaces for machinery and heavy stores such as ammunition. Further, the lower spaces do not provide as wide a tank as could be installed in higher locations. It is evident that any rise in the location of the tank rapidly reduces the effective lever arm and hence the dynamic moment.

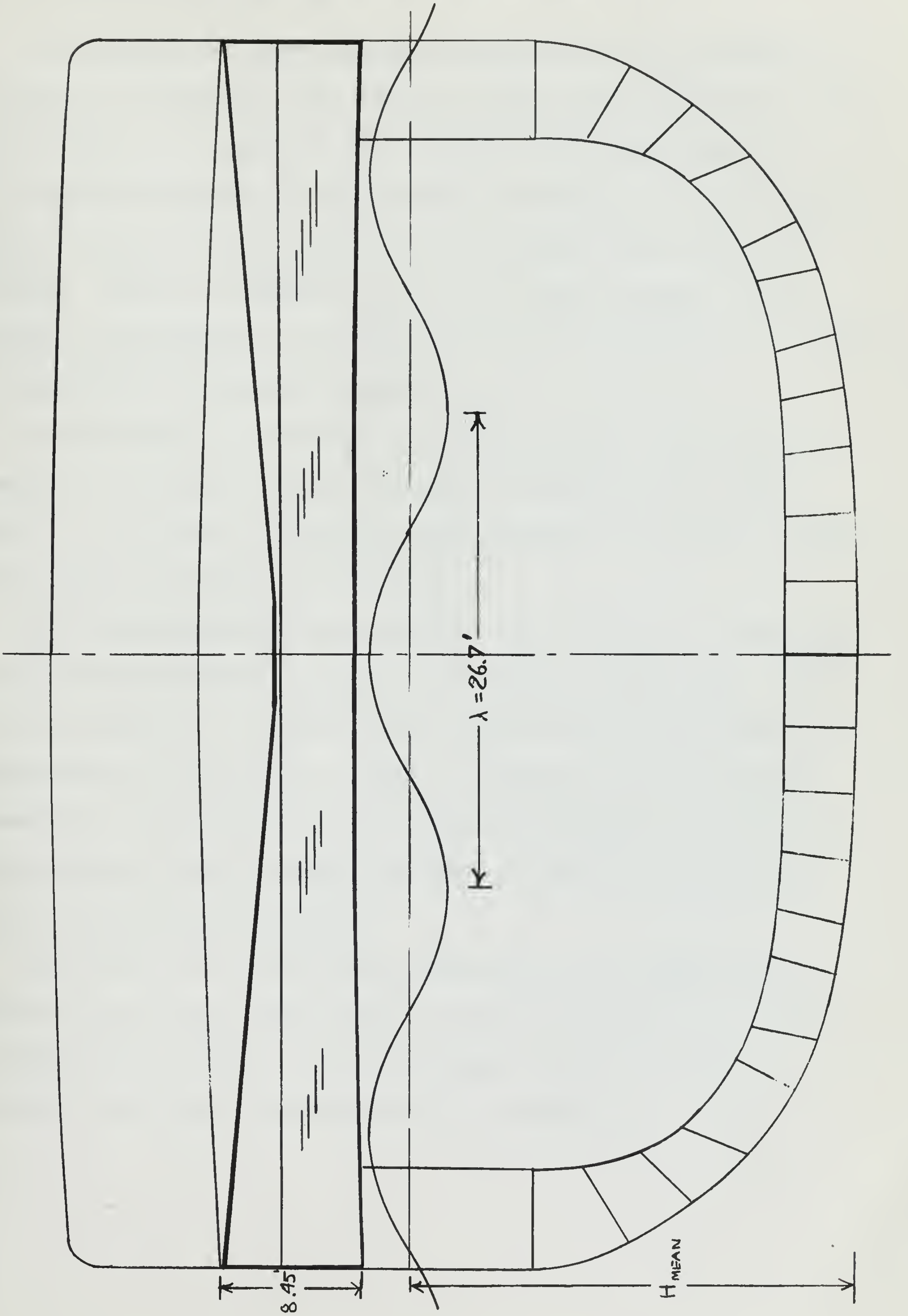
Based on these considerations it is felt that neglect of this factor in the tank calculations is justified. Therefore, the required torque reduction can be considered solely in terms of a reduction in metacentric height due to free surface effect. The desired reduction of metacentric height is one-third of the existing GM. Calculations for the fin system and associated tankage will be found in Appendix A. Location of the tanks is also discussed in the appendix.

It was found that the desired reduction in metacentric height could be accomplished with a tank one deck height high, extending from side to side and with a length of 33.6 feet. One large tank of this length is not envisioned as the ideal installation. Rather, it is expected that the length will be split up into several tanks, longitudinally separated along the ship so as to provide the easiest arrangement, and internally subdivided athwartships so as to permit emptying the oil from a small part when needed, and replacing the oil with ballast water.

A representative section of the proposed tankage is shown in Figure IX.

FIGURE IX

Proposed Free Surface Tanks Installation



D. Intact and Damage Stability

To determine the intact stability characterisitcs it is necessary to calculate the reduction in the righting arm due to the free surface effect. This reduction is righting arm is a function of the angle of heel. It is a simple but arduous task to determine the reduction. The shift of the center of gravity of the liquid at any given angle of heel can be calculated. Thence, the reduction in the ship's righting moment, and consequently, the reduction in the righting arm can be calculated. The method of calculation is presented in Appendix B.

In addition to the reduction in righting arm due to free surface, a reduction is obtained by raising the center of gravity of the liquid. The fuel oil has been relocated in a higher position in the ship to obtain the necessary athwartship tank dimension.

Two tank configurations were studied. One of these had a rectangular cross-section and the other had the top indented in a Vee shape. A comparison of the shift of center of gravity and the resulting reduction in righting arm for the two tanks is made in Figures XV, XVI and XVIII of Appendix B. The Vee-top tank was selected for calculating the effect of free surface on intact and damage stability in order to take advantage of the increased pocketing effect.

The effect of the free surface installation on the damage and intact stability of the typical warship studied was determined by applying the reduction in righting arm to the original righting arm curves. The results are shown in Figures X and XI of Section III.

E. Weight and Space Comparison of Stabilizing Systems

Reference [19] gives a method for estimating the weight of a fin stabilizer system based on data from an existing installation. It is based on the assumption that weight per fin will vary as λ^3, where λ^2 is the ratio of areas per fin in known and new installations. This is believed to be a conservative estimate, since the weight of the operating machinery should not vary by so large a factor.

The following data pertain to the fin stabilizer installation in USS Timmerman, which is stabilized to 5 1/2 degrees at 20 knots. The fin system is essentially the same as that proposed for the ship under investigation, except in the size and number of fins.

Data for USS Timmerman

Displacement (tons)	3409
Stabilization weight (tons, including lost buoyancy)	70.2
Weight of system as percent displacement	2.1
Area of one fin (square feet)	45.0
Weight per fin (tons)	35.1

The ship used in this example has a displacement of 17,685 tons. The area of one fin as calculated in Appendix A is 69.48 square feet. Thus,

$$\lambda^2 = \frac{69.48}{45.0} = 1.54$$

$$\lambda = 1.24$$

$$\lambda^3 = 1.92$$

and the weight of the fin system per fin is,

$$1.92 \times 35.1 = 67.4 \text{ tons.}$$

The weight of the six fin system for the ship without free surface tanks installed will be 404 tons and the weight of the four fin system, excluding weight charged to the tankage, will be 270 tons.

This comparison may not be entirely valid, as it may be assumed that, while the weight of fins, positioning machinery and lost buoyancy will follow this scale effect, the weight of the sensing element and order signal generator will remain nearly constant. The figures are felt to be of the correct order of magnitude, however, representing only a rough approximation.

The weight of the free surface tankage will be only the weight of the athwartship dividing bulkheads and the tank top. The sides of the ship form the tank sides and the existing deck forms the tank bottom. The fluid in the tanks is fuel oil or ballast which would be carried even if no stabilizing system were installed. Although it is unlikely that ballast would be taken on at so high a location, the fuel oil in the tanks represents only 10.8% of the ship's total fuel oil and would probably never be taken from the tanks.

The weight of the bulkheads and tank tops, assuming the length of tankage to be divided into six tanks, and allowing 20.4 $\#/ft^2$ for plating and stiffeners, is about 45 tons. Thus, the total weight of the two sets of fins and the free surface tanks is about 315 tons, a saving of 89 tons over the six fin system.

Chadwick, in Reference [45], referring to fin stabilizers, states that,

> "the percent space requirements will be approximately equal to the percent weight of the system."

In the ship without free surface, the weight of the six fin system

represents 2.28% of the ship's displacement. Then the space required would be 2.28% of the displacement volume, or 14,150 cubic feet.

The weight of the fins in the four fin system represents 1.52 percent of the ship's displacement and the required volume would be 9,430 cubic feet. The tankage consumes a volume of 14,240 cubic feet. Thus, the total volume requirement for the fin and free surface system is 23,670 cubic feet, an increase of 9,250 cubic feet over the six fin system.

If the volume occupied by fuel oil is not charged to the system, on the grounds that this volume would be used for fuel oil regardless of any fin stabilizer system, the chargeable volume is that of the air space above the oil, 4,750 cubic feet. Then, the chargeable volume of the system is 14,180 cubic feet, which is essentially the same volume as that required for the six fin system without free surface.

III. RESULTS

It has been shown that the introduction of a free surface can reduce the requirements of a fin stabilizer system. In the example selected a reduction from six fins to four was made possible by the use of a free surface tank using fuel oil to form the surface. This represents a 22% reduction in overall system weight, somewhat less than a one-third reduction in system cost, since one-third of the machinery is eliminated, and virtually no change in the volume required.

It was determined that the free surface must be an uninterrupted one to avoid the detrimental effects of resonance in the tanks, and that a wrap-around tank of reasonable dimensions would not be compatible with activated fin stabilization.

Table I, below, shows the essential characteristics of the stabilizing systems, one with free surface tanks and one without.

TABLE I

	With Free Surface	Without Free Surface
Capacity	5° at 15 knots	5° at 15 knots
Number of fins	4	6
Rise in center of gravity	1.62 feet	- - -
System weight	315 tons	404 tons
System volume	14,180 ft^3	14,150 ft^3

The effects of the free surface on intact and damage stability are shown in Figures X and XI. Figure XII shows the curve of righting arms at various effective wave slopes and the corresponding

FIGURE X

Righting Arm Curves for Full Load Condition

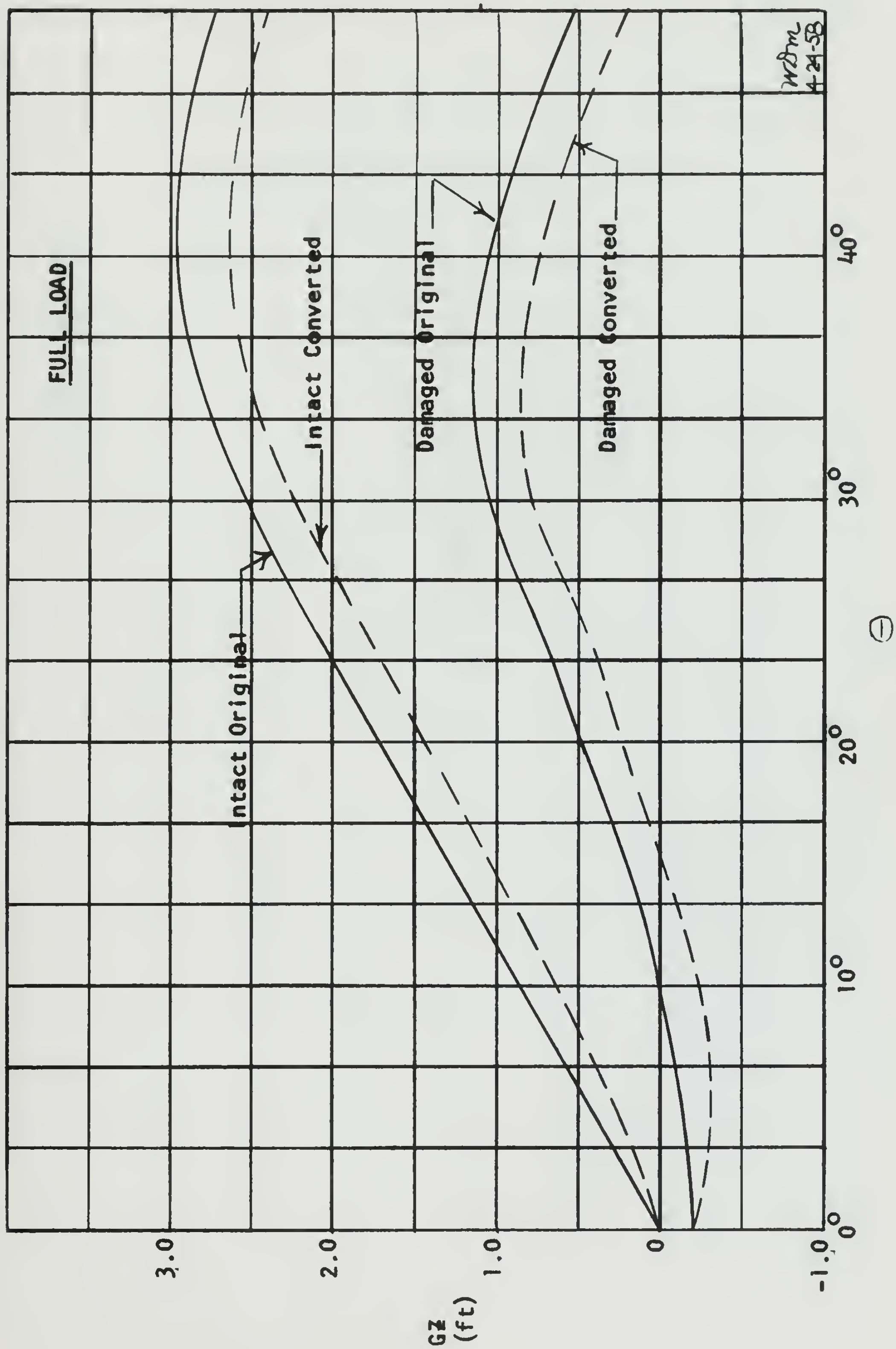


FIGURE XI

Righting Arm Curves for Minimum Operating Condition

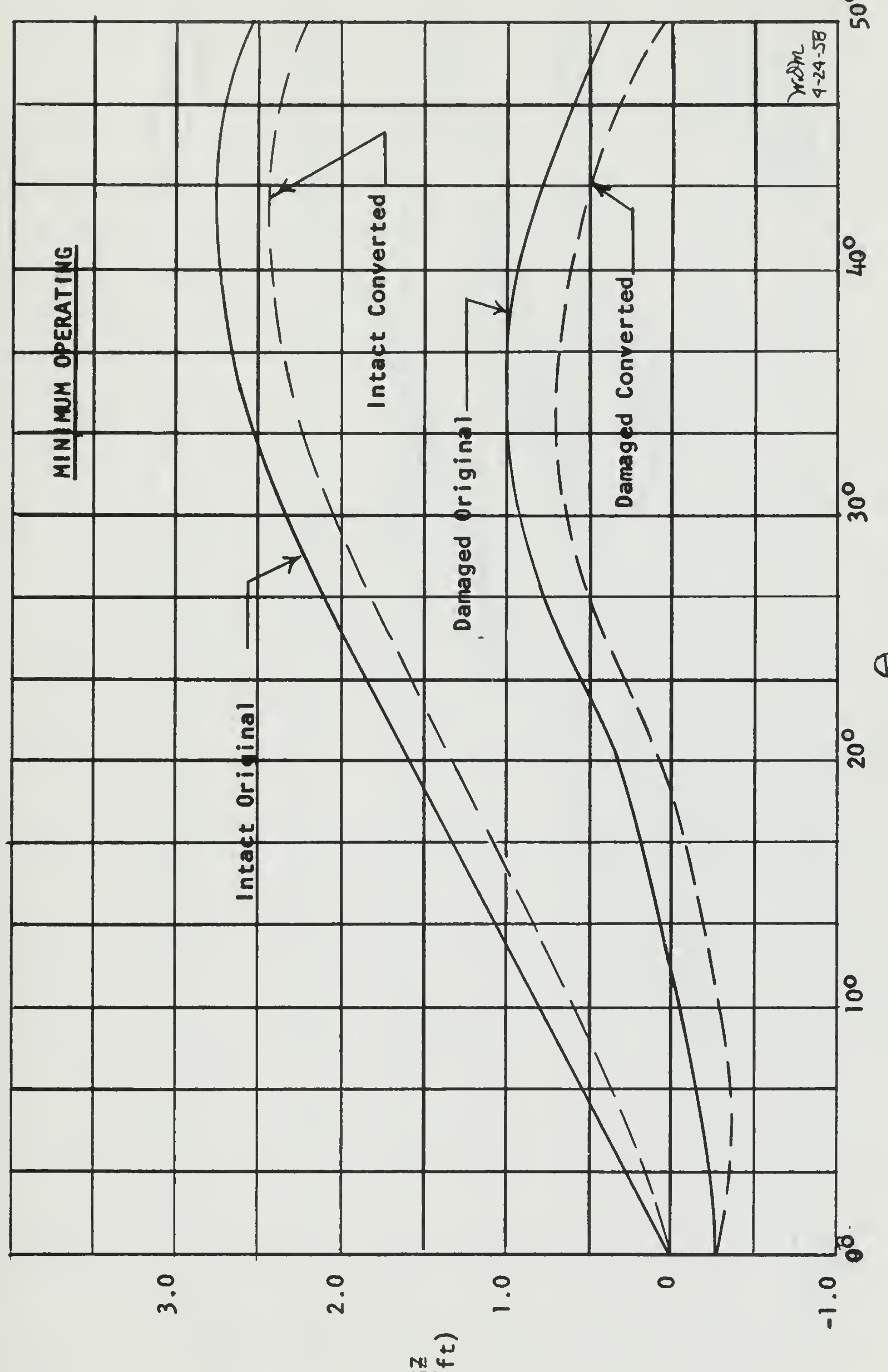
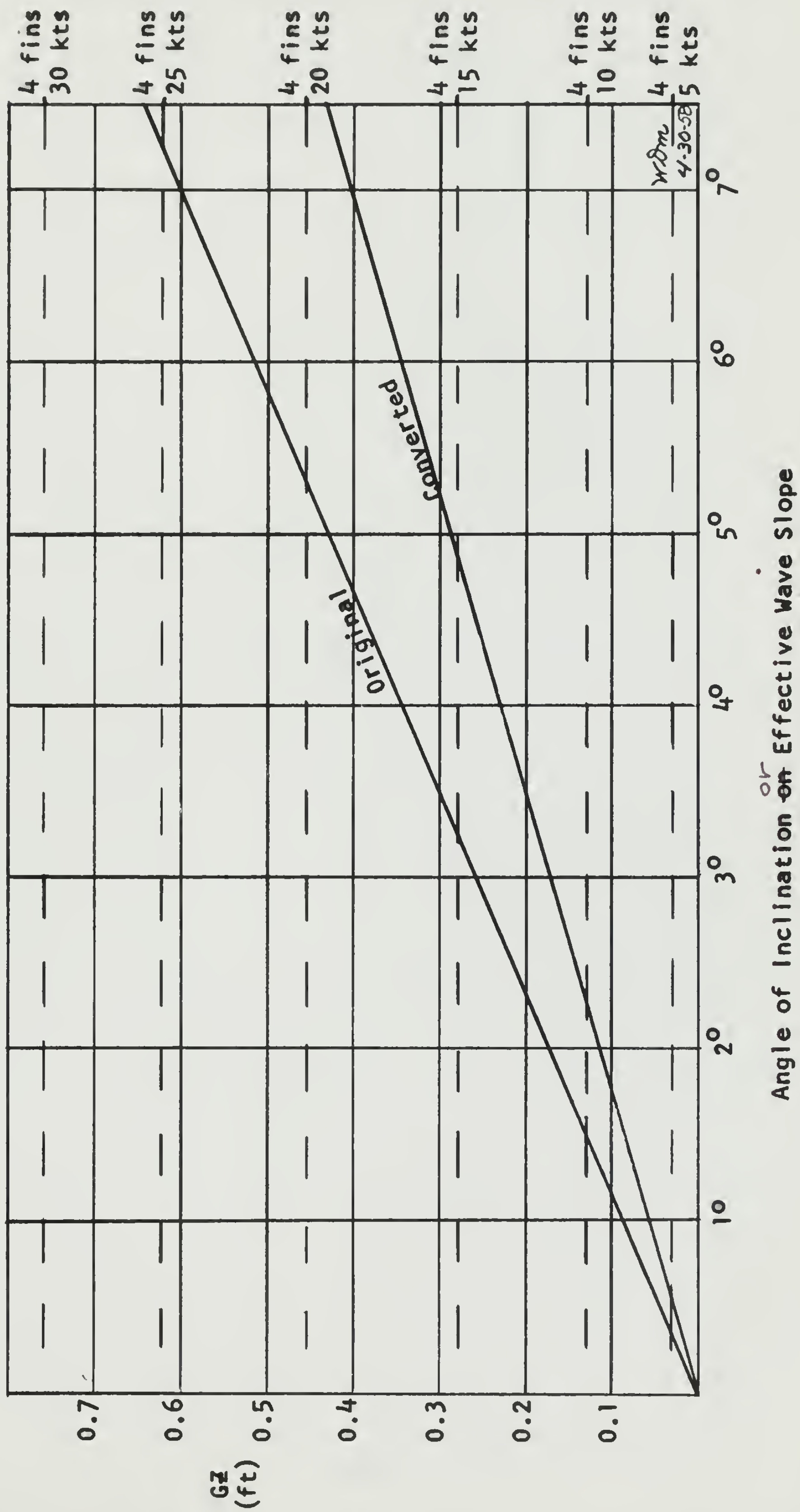


FIGURE XII

Effective Righting Arm of Fin System vs. Buoyant Rigting Arm



effective arm of the fin system at various speeds. The calculations for the effective arm of the fin system will be found in Appendix C.

IV. DISCUSSION OF RESULTS

The free surface tank installation as proposed for the typical warship in this investigation has several disadvantages. Among these are the location of the tanks, the detrimental effects on damage stability and the possibility of violent motions of the fluid during heavy rolling.

Although, the volume of the overall stabilizing system is not appreciably increased by the installation of free surface tanks, it can be seen that the physical location of the tanks within the ship takes up space which is normally used for other functions, which must then be relocated to less desirable areas. This was one of the deciding factors against the Watts water chambers and is generally an undesirable situation.

Further, although the space above the Vee-top of the tanks has not been charged to system volume, this space is of an odd shape and may not be too useful for other purposes.

In the ship used as an example it was necessary to place the tanks at a relatively high point in the ship, in order to obtain a sufficient free surface area without major changes in the ship's machinery installation. This resulted in an actual rise in the ship's center of gravity, or a semi-permanent reduction in metacentric height. It would be desirable to install the tankage so that all of the reduction in metacentric height was accounted for by the free surface effect. This would require a larger tank surface area but would allow restoration of metacentric height simply by disablement of the tank, either by filling completely or by draining completely.

In the installation as proposed, the tanks may be disabled by draining the fluid into lower spaces, such as voids. The free surface effect could thus be nearly eliminated and the center of gravity would also be lowered. However, sufficient empty volume below the tanks must be provided for this purpose, and the volume of the overall system would then be increased. If the tanks were placed low in the ship they could be disabled by filling, with gravity flow, from other reserve fuel oil tanks, thus requiring no added volume.

Some system for disabling the tanks must be provided. There will be occasions when the free surface effect will be undesirable, as, for instance, in very heavy seas when the ship is rolling heavily, or when damage to the ship has occurred.

It can be seen from the curves of righting arm in the most critical damaged condition, Figures X and XI, that a very serious condition of negative metacentric height may exist. Disabling the tanks will greatly alleviate this situation although this particular ship will still have a negative metacentric height.

It should be noted that to disable the tanks, either by draining or filling, requires a positive initiating action, and that, when damage has occurred, conditions may be such that the operating positions for this action are inaccessible, due to fire of flooding. Further, the operating mechanism, valves, etc., may be damaged or the personnel assigned to disable the tanks may be incapacitated. Thus, the existence of the free surface may be a real hazard to the ship.

Some reduction in the free surface effect is obtained by the pocketing effect of the Vee-top tank if the vessel should list to

7^{o}. This feature is advantageous to damage stability, but may not be advantageous in the intact condition when the ship is rolling heavily.

If the sea conditions are such that performance of the stabilizing system is low, or if the fin system is inoperative, fairly heavy rolling may occur. Then it may be expected that violent motions of the fluid in the tanks will build up, even to the extent of the formation of breaking waves within the tank. Such phenomena were observed in the Watts water chambers installed in HMS Inflexible. It may be expected that, in heavy rolling, the fluids would slam against the tank top with considerable force, resulting in noise and vibrations that may be unacceptable. Here, again, it would be desirable to disable the tanks.

V. CONCLUSIONS AND RECOMMENDATIONS

Many of the disadvantages of the free surface system presented in Section IV are a direct result of the attempt to adapt the system to an existing ship. For example, the location of the tanks in choice spaces, the resultant rise of the ship's center of gravity and the dangerous damage stability condition relate directly to the typical warship selected for this study. The drawbacks of the installation outweigh the small improvements made in weight and cost of the stabilizing system by so much that this system is not considered practical for this ship. Similar drawbacks would be encountered in the application of the system to other existing ships.

It is felt, however, that, if the design of the system were considered in conjunction with the development of plans for a new ship, suitable compromises could be reached, eliminating most of the disadvantages.

The impairment of damage stability is considered the critical factor in any decision to install this type of system. Such a decision is the function of the design agency, which must comply with the standards of stability imposed upon it.

The future applicability of this type of system will not be great. The necessity for such a system is generally limited to warships which must have a large metacentric height. It is expected that the majority of warships built in the future will be nuclear powered and therefore will not carry large quantitites of fuel oil which might be used for free surface tankage.

[illegible]

V.I APPENDIX

APPENDIX A

1. Calculation of Fin System for Vessel with No Free Surface Tanks Installed

The characterisitcs of the vessel selected for the example are as follows:

Displacement, full load	17,685 tons
Length between perpendiculars	664 feet
Breadth, extreme	70 feet, 9-3/4 inches
Draft, full load	24.68 feet
Height of metacenter above keel	31.96 feet
Height of center of gravity	27.11 feet
Metacentric height (corrected)	4.85 feet
Height of center of buoyancy	14.50 feet

The axis of rolling is assumed halfway between the center of buoyancy and the center of gravity. The fins are located in a line from the axis of roll through the turn of the bilge. Depth of immersion is 18 feet. Doubly all movable fins will be used.

Reference [19] states,

"For doubly all movable fins having a geometric aspect ratio between 1.75 and 2.25 it is possible to attain a lift coefficient of 1.70 at a fin angle of 20 degrees when there is no cavitation ".

Cavitation number,

$$\sigma = \frac{p_o - e}{1/2 \rho V^2} \qquad \text{(A-1)}$$

where p_o is the static pressure at the depth of the fins,

e is the vapor pressure of the fluid, taken for a temperature of 50° F,

ρ is the mass density of the fluid

U is the relative speed of the fluid at the location of the fins, and is taken to be the forward speed of the ship.

[illegible]

For this ship the cavitation number is

$$\sigma = \frac{51.0 - 0.41}{1/2 \times 2 \times 2.85\, V_k^2} = \frac{1135}{V_k^2}$$

$$V_k = 15 \text{ kts}, \quad \sigma = 5.05$$

Figure 6 of reference [19] shows curves of lift coefficients for various cavitation numbers for a maximum lift coefficient of 1.70. From this curve, at the given cavitation number, the maximum lift coefficient is 1.63.

The equation for determination of fin stabilizer requirements is:

$$\Delta\, GM \sin \psi = 1/2\, \rho\, C_L A V^2 \times 2a \qquad \text{(A-2)}$$

where Δ is the ship's displacement in pounds

GM is the ship's metacentric height in feet

ψ is the capacity of the system

ρ is the mass density of the fluid

C_L is the lift coefficient

A is the area of the fins on one side

V is the velocity of the fluid at the fins, ft./sec.

a is the lever arm at the center of pressure of the fins

Δ = 17,685 X 2240 = 39,614,400 lb.

GM = 4.85 ft.

$\psi = 5^\circ$, $\sin \psi = .0871$

$\rho = 2$

$C_L = 1.63$

A is to be determined

V = 1.688 X 15 = 25.3 ft./sec.

a = 38.5 feet (based on an assumed span of 12 feet)

Substituting the above values in equation (A-2) the area of fins on one side is 208.33 square feet. With the assumed span length of 12 feet the total chord length must be 17.36 feet. If three sets of fins are installed the chord length of each fin will be 5.79 feet. This results in an aspect ratio of 2.07, which is within the band of aspect ratios which will produce the desired lift coefficient.

2. Calculation of Fin System for Use in Conjunction with Free Surface

For this system the ship's metacentric height has been arbitrarily reduced by one-third. $G_vM = 2/3\ GM = 3.23$ feet.

The same type of fins will be used and all pertinent values are taken from the preceding calculation for fins with no free surface. Applying these values to equation (A-2) with the revised metacentric height the area of fins on one side is 138.74 square feet. With the same span length of 12 feet the total chord length is 11.56 feet. If two pairs of fins are installed the chord length per fin will be 5.78 feet, resulting in an aspect ratio of 2.08, which is again withing the acceptable limits.

Therefor the fin system to be used with the free surface installation will consist of two pairs of fins, each having a span of 12 feet and a chord length of 5.78 feet. To all intents and purposes these fin sets will be identical to those used in the ship with no

free surface, the only difference being that two sets are used instead of three.

3. Calculations for Tankage to Establish Free Surface Effect

The free surface tank must destory one-third of the ship's metacentric height. This will amount to 1.62 feet. The inertia of such a tank must be quite large and can best be obtained by a wide tank. To keep size to a minimum it was decided to use a tank which extends from one side of the ship to the other. A study of the plans of the ship under investigation showed that such an installation could not be accomplished at a location low in the ship so the tankage will be installed in the deck just above the machinery spaces.

A mean width of tank was established by taking an average based on cubes of ship width at various suitable tank locations.

$$
\begin{array}{lll}
b_1 = 70.0 & \quad & b_1^3 = 343{,}000 \\
b_2 = 65.5 & \quad & b_2^3 = 281{,}011 \\
b_3 = 70.5 & \quad & b_3^3 = \underline{350{,}403} \\
 & & \qquad\; 974{,}414
\end{array}
$$

$$b_m^3 = 324{,}805$$

$$b_m = 68.75 \text{ feet} \quad \text{(mean width of tank)}$$

In this installation the transfer of fuel oil to tanks higher in the ship than those in which fuel is normally carried will cause a rise in the center of gravity which must be taken into account in the reduction of metacentric height. If it were possible to install

a tank system such that there were no actual rise in the center of gravity the tank system would have to be somewhat larger than the one proposed herein.

The center of gravity of all fuel oil or ballast tankage presently existing is located 11.78 feet above the base line. The lower deck of the space intended for free surface tankage is 28 feet above the base line.

The reduction in metacentric height may be expressed as a rise in the center of gravity, consisting of both a real change and a virtual change due to the free surface.

$$GG_v = \frac{\frac{lbd}{v_o}\left(28 + d/2 - 11.78\right)}{\Delta} + \frac{v_w}{v_o}\,\frac{lb^3}{12\,V} \qquad \text{(A-3)}$$

where b is the mean width of tank, 68.75 feet

d is the depth of fluid in the tank, 4.22 feet

l is the length of tank, to be determined

V is the volume of displacement of the ship, 618,975 ft^3

v_o is the specific volume of fuel oil, 38 ft^3/ton

v_w is the specific volume of sea water, 35 ft^3/ton

Substitution of the above values into equation (A-3) results in a tank length of 33.6 feet. It should be noted that the inertia effect includes a factor for the relative densities of oil and water. The tankage must be sized on the assumption that oil is in the tanks as this results in a larger tank than would be required if water were in the tanks.

It should also be noted that the depth of oil is just one half

of the height of the tank. With this amount of fluid in the tank the pocketing effect will begin to reduce the free surface when the angle of inclination of the fluid surface with respect to the ship is 7 degrees. The capacity of the stabilizing system is 5 degrees. If the motion of the ship is such that fluid motions exceed this value, the capacity of the stabilizer system will be exceeded and it will be desirable to restore as much of the metacentric height as possible. This would be particularly true in the event of a permanent list such as might be caused by damage to the ship.

To enhance the pocketing effect at angles above 7 degrees it was decided to form the tank top in the shape of a Vee, so that the fluid goes hard up against the tank top on one side at an angle of 7 degrees.

The volume of fuel oil required will be

$$lbd = 9760 \text{ ft}^3$$

and the corresponding weight of fuel will be 256.8 tons. Thus, the actual shift in the center of gravity will be

$$\frac{(256.8)(18.33)}{17,685} = .266 \text{ feet.}$$

The virtual shift in the center of gravity will be

$$\frac{35 \times 324805 \times 33.6}{38 \times 12 \times 618,975} = 1.354 \text{ feet.}$$

Inasmuch as the volume above the Vee shaped tank may be recoverable for other purposes, the volume of tankage required is

$$3/2 \times 9760 = 14,240 \text{ ft}^3$$

This ship normally carried 2,379.4 tons of fuel oil, 385.5 tons of it in fuel oil or ballast tanks. The volume of oil required will be easily obtained.

The proposed tankage is shown in Figure IX.

APPENDIX B

1. In the stability investigation of the vessel with the proposed free surface tank the location of the center of gravity of liquid in the tank, for various angles of heel, is required.

2. The location of the center of gravity of liquid at angles equal to or less than seven degrees is arrived at as follows:

Referring to Figure XIII, representing angles of heel up to and including seven degrees the following notation is used:

h_s = depth of liquid in tank at low side of ship

h_p = depth of liquid in tank at high side of ship

b = athwartship dimension of Vee-top tank

$\bar{x}$ = athwartship distance from tank boundary at low side of ship to center of gravity of fluid

$\bar{y}$ = distance from bottom of tank parallel to vertical center-line fo ship to center of gravity of fluid

θ = angle of heel of vessel

By dividing the cross-sectional area of the liquid into simple geometric shapes equations (B-1) and (B-2), locating the center of gravity of the liquid, are derived.

$$\bar{x} = \frac{(h_p b) \cdot b/2 + \frac{1}{2} b\,(h_s - h_p) \cdot (b/3)}{\frac{1}{2} b\,(h_s + h_p)} \qquad \text{(B-1)}$$

$$\frac{(h_p b)\, h_p/2 + \frac{1}{2} b\,(h_s - h_p) \left[\frac{(h_s - h_p)}{3} + h_p \right]}{\frac{1}{2} b\,(h_s + h_p)} \qquad \text{(B-2)}$$

The values of the variable dimensions h_s and h_p are determined for angles up to 7 degrees by simple trigonometric relationships.

FIGURE XIII

For Angles of Heel Up to and Including Seven Degrees

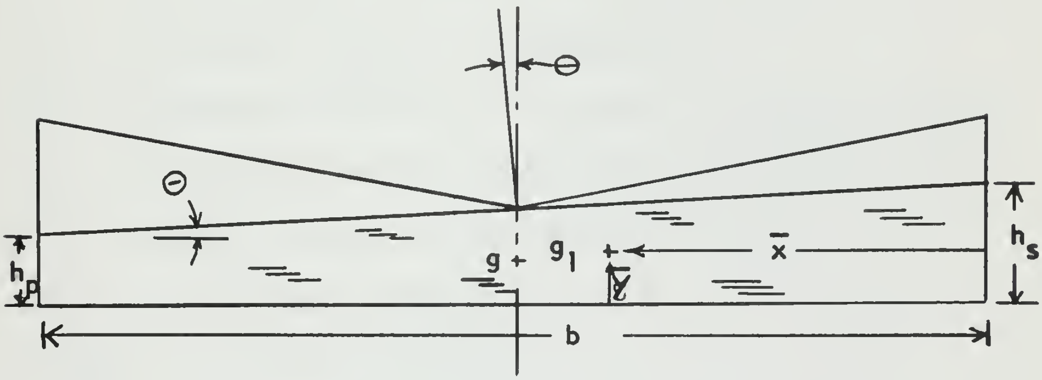
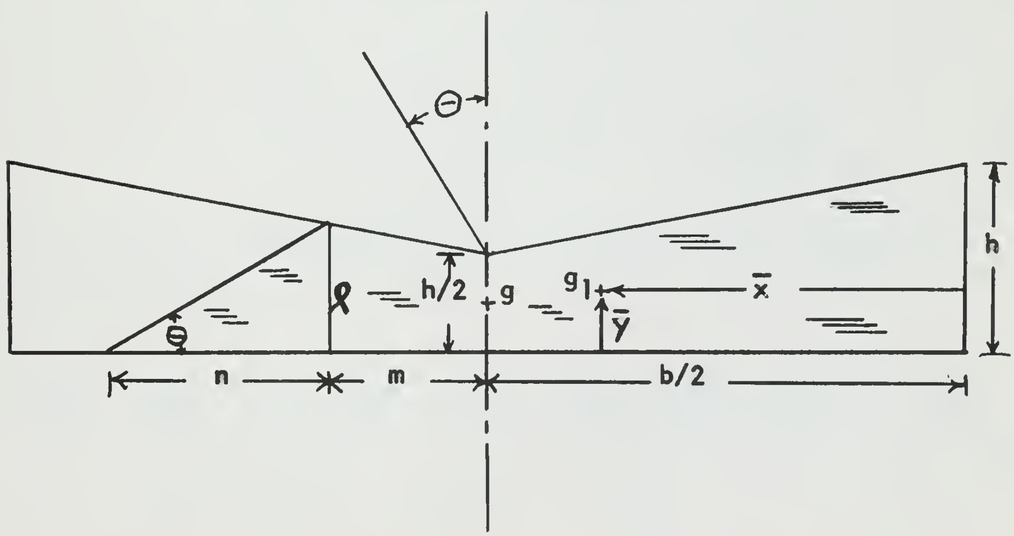


FIGURE XIV

For Angles of Heel Greater than Seven Degrees



3. At inclinations greater than seven degrees the liquid is hard up against the tank top on the low side of the vessel. Referring to Figure XIV, representing angles of heel greater than seven degrees, it is seen that the cross-section is divided into simple geometric shapes to be used in locating the center of gravity of the liquid. The following notation is used:

h = depth of liquid in tank at low side of ship

b = athwartship dimension of Vee-top tank

m = athwartship dimension of trapezoid on high side of ship

n = athwartship dimension of triangle on high side of ship

ℓ = dimension common to trapezoid and triangle

θ = angle of heel of vessel

$\bar{x}$ and $\bar{y}$ as defined previously

At all inclinations after pocketing, i.e., 7 degrees, the cross-section area of liquid on the high side of the tank centerline is constant. Therefore,

$$A_{\triangle} + A_{\text{trap}} = \text{constant} \tag{B-3a}$$

$$\frac{1}{2} \cdot \frac{b}{2} \cdot \frac{h}{2} = \text{constant} \tag{B-3b}$$

$$A_{\triangle} + A_{\text{trap}} = \frac{1}{2} \cdot \frac{b}{2} \cdot \frac{h}{2} \tag{B-3c}$$

$$\frac{1}{2} \cdot \frac{b}{2} \cdot \frac{h}{2} = \frac{1}{2} n\ell + m\left(\frac{\ell + \frac{h}{2}}{2}\right) \tag{B-3d}$$

The following equations define the location of the center of gravity of the liquid:

$$\bar{x} = \frac{\frac{hb^2}{16} + \frac{hb^2}{48} + \frac{mh}{2}\left(\frac{b}{2}+\frac{m}{2}\right) + \frac{m}{2}\left(\ell - \frac{h}{2}\right)\left(\frac{b}{2} + \frac{2m}{3}\right) + \frac{n\ell}{2}\left(\frac{b}{2} + m + \frac{n}{3}\right)}{\frac{b}{2}\left(\frac{h + h/2}{2}\right) + \frac{1}{2} m\left(\frac{h}{2} + \ell\right) + \frac{n\ell}{2}} \tag{B-4}$$

$$\bar{y} = \frac{\frac{bh^2}{16} + \frac{bh}{8}\left(\frac{h}{6} + \frac{h}{2}\right) + \frac{mh^2}{8} + \frac{1}{2}m\left(\ell - \frac{h}{2}\right)\left[\frac{\left(\ell - \frac{h}{2}\right)}{3} + \frac{h}{2}\right] + \frac{n\ell^2}{6}}{\frac{b}{2}\left(\frac{h + h/2}{2}\right) + \frac{1}{2}m\left(\frac{h}{2} + \ell\right) + \frac{n\ell}{2}} \tag{B-5}$$

For the Vee-top tank proposed in Appendix A the fixed values of h and b are,

$$h = 8.45 \text{ feet}, \qquad b = 68.75 \text{ feet}$$

These values of h and b result in the following relationships, by geometry, between the unknowns

$$n = \frac{1182 - 68.75\, m - m^2}{34.375 + m} \tag{B-6}$$

$$\ell = 4.225 + 0.123\, m \tag{B-7}$$

Selection of arbitrary values of the dimension m yields corresponding values for n and ℓ, by substitution in equations (B-6) and (B-7). The corresponding angle of heel is readily determined by trigonometry,

$$\theta = \tan^{-1}\left(\frac{\ell}{n}\right) \tag{B-8}$$

Substitution of m, n, and ℓ into equations (B-4) and (B-5) yields values for $\bar{x}$ and $\bar{y}$, respectively, at the angle of inclination, θ.

4. The values of $\bar{x}$ and $\bar{y}$, which are measured from the tank boundaries, are used in the determination of $\delta\bar{x}$ and $\delta\bar{y}$, which represent the shift in the center of gravity coordinates for various angles of heel. See Figures XV and XVI.

5. The reduction in righting arm of vessel is made up of two components. These components are due to the shift of the center of gravity of the vessel by relocating the oil in the high position of

FIGURE XV

Horizontal Shift in Center of Gravity of a Partially Filled Tank vs. Angle of Heel

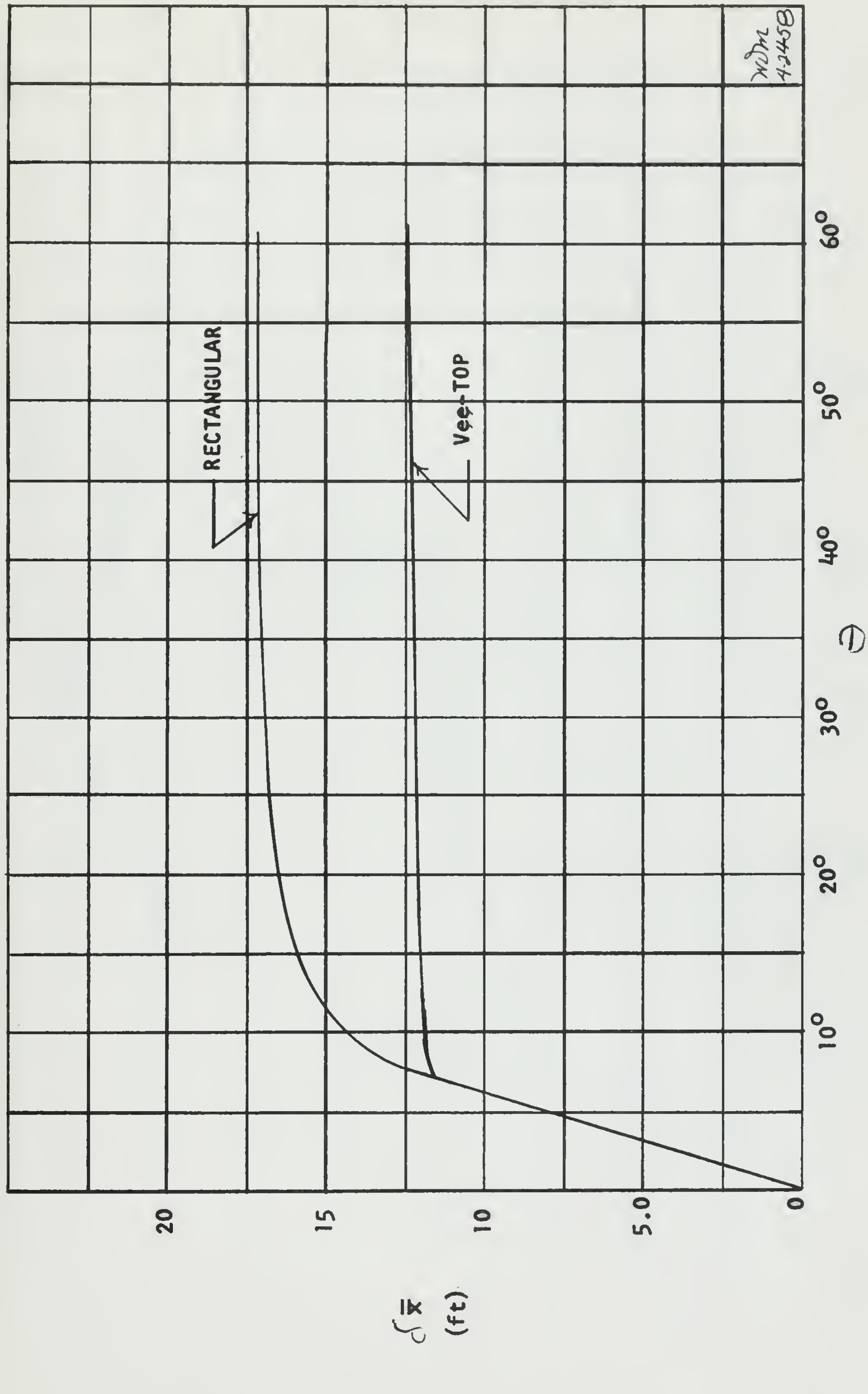
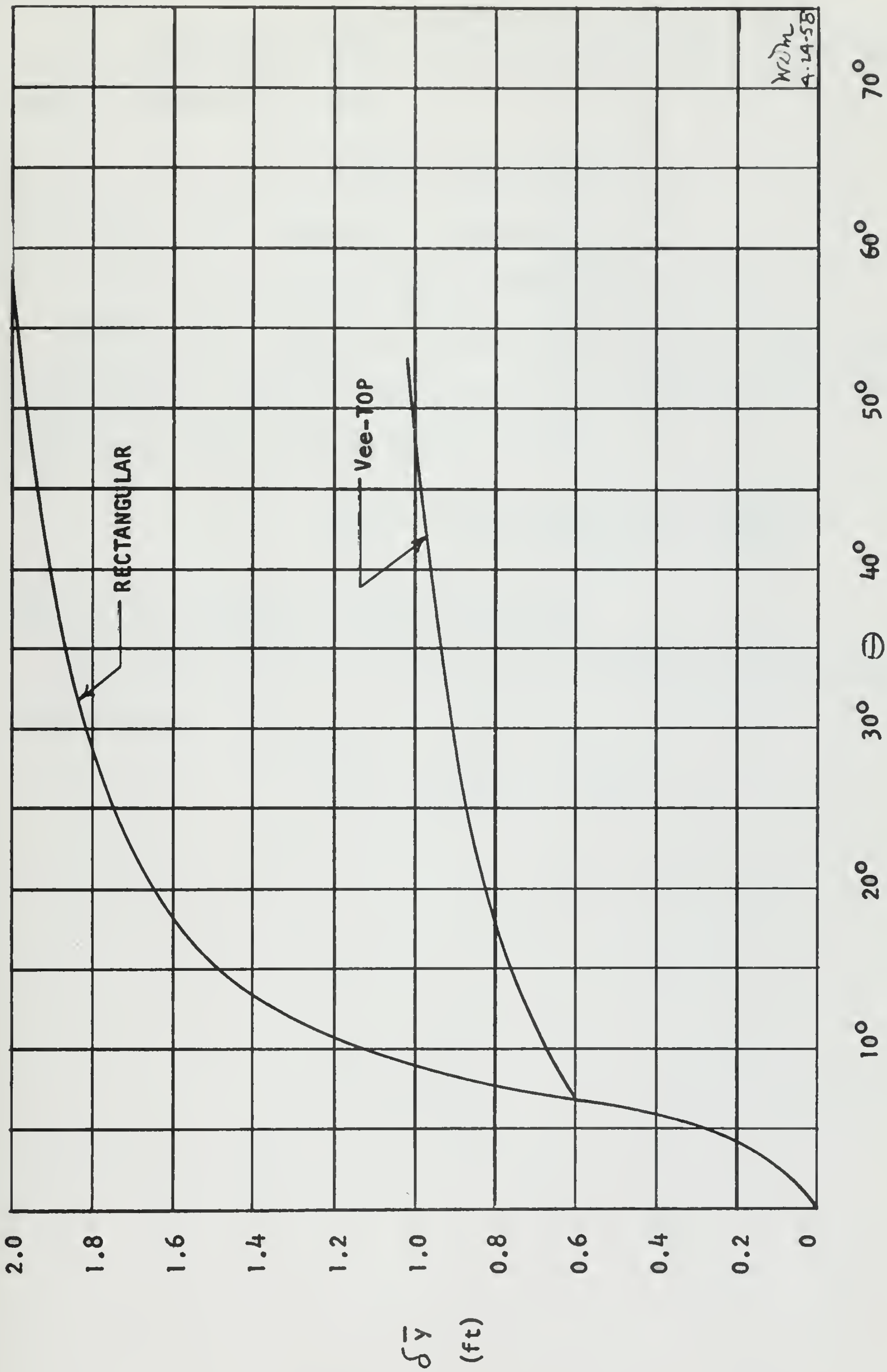


FIGURE XVI

Vertical Shift in Center of Gravity of a Partially Filled Tank vs. Angle of Heel



the tanks and the motion of the oil within the tanks when the vessel is inclined.

The reduction in righting arm due to the vertical rise of the ship's center of gravity is simply,

$$\delta GZ_o = GG_1 \sin\theta \qquad \text{(B-9)}$$

where δGZ_o is the reduction in righting arm

GG_1 is the vertical rise in the center of gravity

Referring to Figure XVII it can be seen that the moment caused by the shift of weight of liquid due to the inclination may be expressed as a reduction in righting arm, δGZ_1.

$$\delta GZ_1 = \frac{W(\delta\bar{x}\cos\theta + \delta\bar{y}\sin\theta)}{\Delta} \qquad \text{(B-10)}$$

where W is the weight of fluid in the tank.

The total reduction in righting arm with the proposed Vee-top tank is then,

$$\delta GZ = \delta GZ_o + \delta GZ_1 \qquad \text{(B-11)}$$

FIGURE XVII

Diagram Used in Calculating Reduction of Righting Arm Due to Free Surface

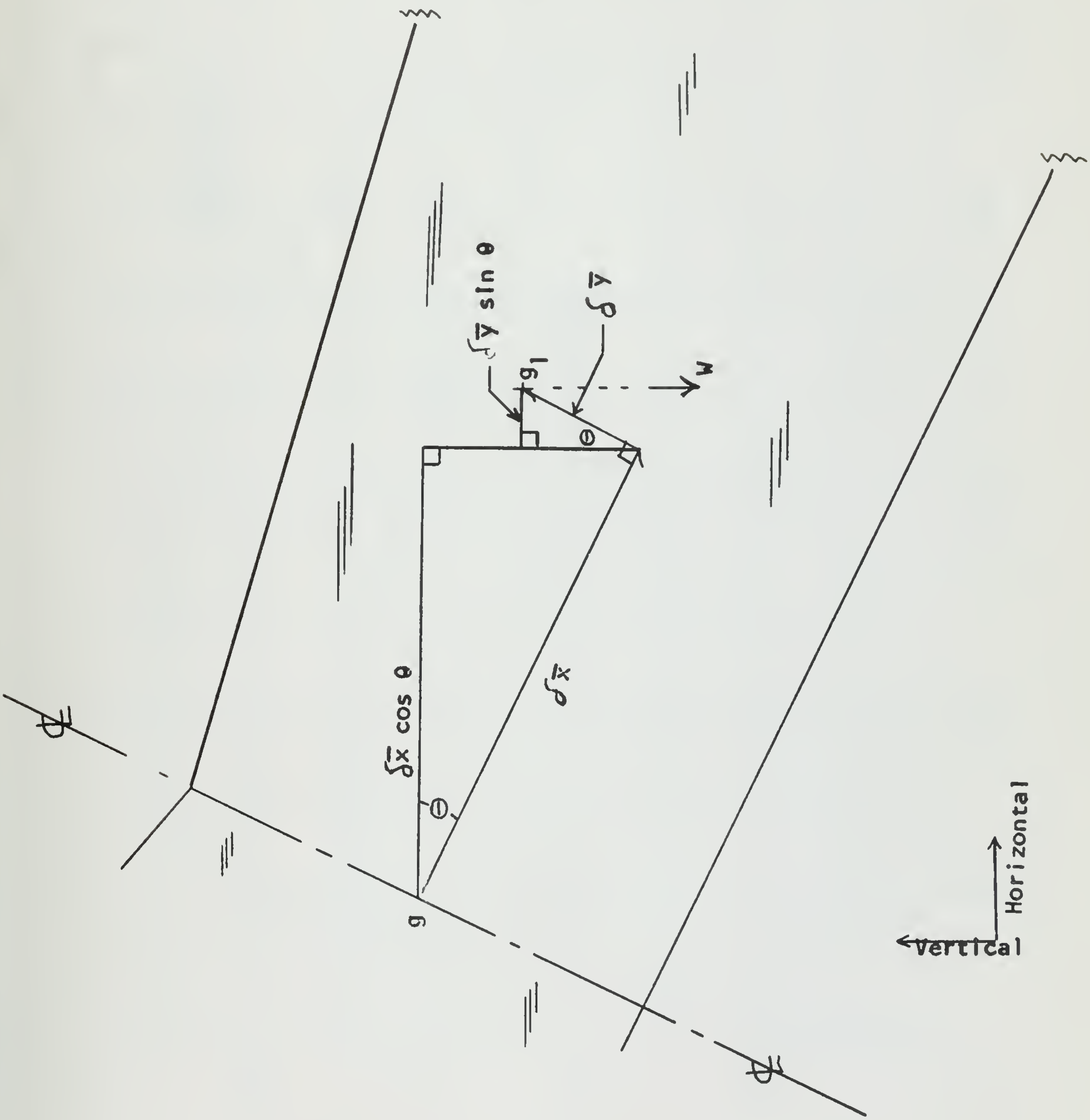
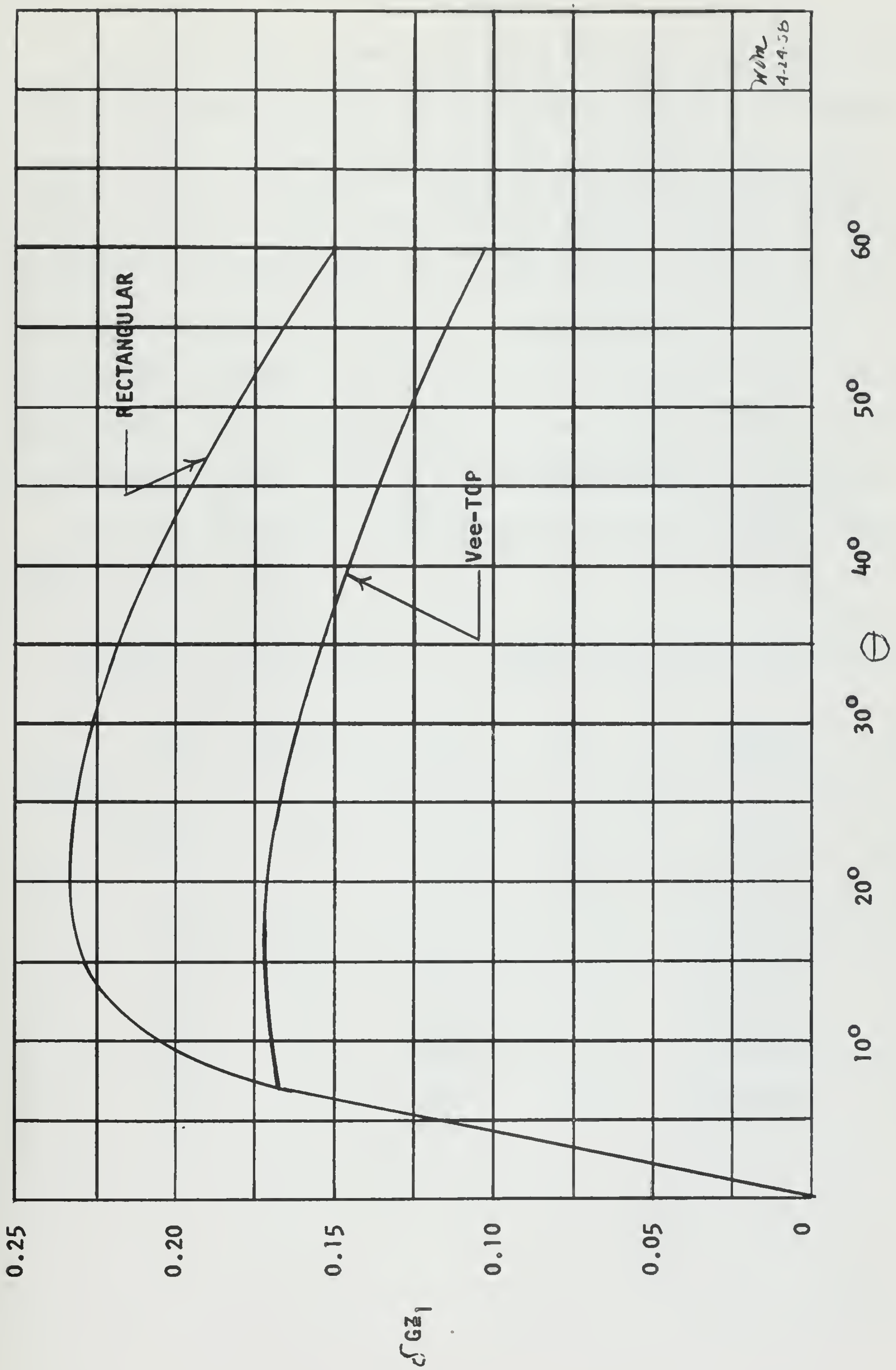


FIGURE XVIII

Reduction in RightingArm vs. Angle of Heel
Due Solely to Free Surface Effect



APPENDIX C

As an indication of the stabilizing ability of the proposed system the torque producing ability of the fins was calculated at various speeds. This torque was then divided by the ship's displacement to yield an effective moment arm of the fin system, which was then plotted in Figure XII, superimposed on the righting arm curves of the ship. The intersection of the fin effective arm line with the righting arm curve shows the capacity of the fin system at a given speed. To indicate the capacity of the system without the free surface tankage installed the curve of righting arms for the unconverted ship is also shown.

The calculations for effective arm of the fin system are shown below in tabular form

$$\text{Torque} = \frac{1}{2} C_L A V^2 \times 2a = 10683\ C_L V^2 \text{ (foot pounds)} \qquad \text{(C-1)}$$

V_k (knots)	σ	C_L max	V (ft/sec)	V^2	Torque (ft-ton)	Arm, a. (ft)
5	45.45	1.7	8.43	69.1	560.24	.0317
10	11.35	1.7	16.88	285	2,310.68	.1307
15	5.05	1.63	25.3	640	4,975.22	.2813
20	2.84	1.48	33.8	1140	8,046.59	.4550
25	1.82	1.30	42.2	1775	11,004.92	.6223
30	1.26	1.10	50.6	2560	13,430.06	.7594

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Abbreviations Used Throughout the Reference List:

INA	Institution of Naval Architects
SNAME	Society of Naval Architects and Marine Engineers
ASNE	American Society of Naval Engineers
ASME	American Society of Mechanical Engineers